Kansas College and Career Ready Standards For Mathematics

Flip Book

Grade 5

Updated Fall 2014

This project used the work done by the Departments of Educations in Ohio, North Carolina, Georgia, engageNY, NCTM, and the Tools for the Common Core Standards.

About the Flip Books

The development of the "flip books" is in response to the adoption of the Common Core State Standards by the state of Kansas in 2010. Teachers who were beginning the transition to the new Kansas Standards—Kansas College and Career Ready Standards (KCCRS) needed a reliable starting place that contained information and examples related to the new standards.

This project attempts to pull together, in one document some of the most valuable resources that help develop the intent, the understanding and the implementation of the KCCRS. The intent of these documents is to provide a starting point for teachers and administrators to begin unraveling the standard and is by no means the only necessary or complete resource that supports implementation of KCCRS.

This project began in the summer 2012 with the work of Melisa Hancock (Manhattan, KS), Debbie Thompson (Wichita, KS) and Patricia Hart (Wichita, KS) who provided the initial development of the "flip books". The "flip books" are based on a model that Kansas had for earlier standards however, this edition is far more comprehensive than those in the past. The current editions incorporate the resources from: other state departments of education, documents such as the content progressions, and other reliable sources including The National Council of Teachers of Mathematics and the National Supervisors of Mathematics. The current product was a compilation of work from the project developers in conjunction with many mathematics educators from around the state. In addition, mathematics educators across the country have suggested changes/additions that could or should be made to further enhance its effectiveness. The document is posted on the KATM website at www.katm.org and will continue to undergo changes periodically. When significant changes/additions are implemented the necessary modification will be posted and dated.

The initial development of the current update to the "flip books" was driven by the need expressed by teachers of mathematics in Kansas and with the financial support from Kansas Department of Education and encouragement from the Kansas Association of Teachers of Mathematics. These "flip books" have become an ongoing resource that will continue to evolve as more is learned about high quality instruction for the KCCRS for mathematics.

For questions or comments about the flipbooks please contact Melisa Hancock at melisa@ksu.edu.

Planning Advice--Focus on the Clusters

The (mathematics standards) call for a greater focus. Rather than racing to cover topics in today's mile-wide, inch-deep curriculum, we need to use the power of the eraser and significantly narrow and deepen how time and energy is spent in the mathematics classroom. There is a necessity to focus deeply on the major work of each grade to enable students to gain strong foundations: solid conceptually understanding, a high degree of procedural skill and fluency, and the ability to apply the mathematics they know to solve problems both in and out of the mathematics classroom. (www.achievethecore.org)

As the Kansas College and Career Ready Standards (KCCRS) are carefully examined, there is a realization that with time constraints of the classroom, not all of the standards can be done equally well and at the level to adequately address the standards. As a result, priorities need to be set for planning, instruction and assessment. "Not everything in the Standards should have equal priority" (Zimba, 2011). Therefore, there is a need to elevate the content of some standards over that of others throughout the K-12 curriculum.

When the Standards were developed the following were considerations in the identification of priorities: 1) the need to be qualitative and well-articulated; 2) the understanding that some content will become more important than other; 3) the creation of a focus means that some essential content will get a greater share of the time and resources "While the remaining content is limited in scope." 4) a "lower" priority does not imply exclusion of content but is usually intended to be taught in conjunction with or in support of one of the major clusters.

"The Standards are built on the progressions, so priorities have to be chosen with an eye to the arc of big ideas in the Standards. A prioritization scheme that respects progressions in the Standards will strike a balance between the journey and the endpoint. If the endpoint is everything, few will have enough wisdom to walk the path, if the endpoint is nothing, few will understand where the journey is headed. Beginnings and the endings both need particular care. … It would also be a mistake to identify such standard as a locus of emphasis. (Zimba, 2011)

The important question in planning instruction is: "What is the mathematics you want the student to walk away with?" In planning for instruction "grain size" is important. Grain size corresponds to the knowledge you want the student to know. Mathematics is simplest at the right grain size. According to Daro (Teaching Chapters, Not Lessons—Grain Size of Mathematics), strands are too vague and too large a grain size, while lessons are too small a grain size. About 8 to 12 units or chapters produce about the right "grain size". In the planning process staff should attend to the clusters, and think of the standards as the ingredients of cluster, while understanding that coherence exists at the cluster level across grades.

A caution--Grain size is important but can result in conversations that do not advance the intent of this structure. Extended discussions that argue 2 days instead of 3 days on a topic because it is a lower priority detract from the overall intent of suggested priorities. The reverse is also true. As Daro indicates, lenses focused on lessons can also provide too narrow a view which compromises the coherence value of closely related standards.

The video clip Teaching Chapters, Not Lessons—Grain Size of Mathematics that follows presents Phil Daro further explaining grain size and the importance of it in the planning process. (Click on photo to view video.)

Along with "grain size", clusters have been given **priorities** which have important implications for instruction. These priorities should help guide the focus for teachers as they determine allocation of time for both planning and instruction. The priorities provided help guide the focus for teachers as they demine distribution of time for both planning and instruction, helping to assure that students really understand before moving on. Each cluster has been given a priority level. As professional staffs begin planning, developing and writing units as Daro suggests, these priorities provide guidance in assigning time for instruction and formative assessment within the classroom.

Each cluster within the standards has been given a priority level by Zimba. The three levels are referred to as:—**Focus**, **Additional** and **Sample**. Furthermore, Zimba suggests that about 70% of instruction should relate to the Focus clusters. In planning, the lower two priorities (Additional and Sample) can work together by supporting the Focus priorities. The advanced work in the high school standards is often found in "Additional and Sample clusters". Students who intend to pursue STEM careers or Advance Placement courses should master the material marked with "+" within the standards. These standards fall outside of priority recommendations.

Recommendations for using cluster level priorities

Appropriate Use:

- Use the priorities as guidance to inform instructional decisions regarding time and resources spent on clusters by varying the degrees of emphasis
- Focus should be on the major work of the grade in order to open up the time and space to bring the Standards for Mathematical Practice to life in mathematics instruction through: sense-making, reasoning, arguing and critiquing, modeling, etc.
- Evaluate instructional materials by taking the cluster level priorities into account. The major work of the grade must be presented with the highest possibility quality; the additional work of the grade should indeed support the Focus priorities and not detract from it.
- Set priorities for other implementation efforts taking the emphasis into account such as: staff development; new curriculum development; revision of existing formative or summative testing at the state, district or school level.

Things to Avoid:

- Neglecting any of the material in the standards rather than connecting the Additional and Sample clusters to the other work of the grade
- Sorting clusters from Focus to Additional to Sample and then teaching the clusters in order. To do so would remove the coherence of mathematical ideas and miss opportunities to enhance the focus work of the grade with additional clusters.
- Using the clusters' headings as a replacement for the actual standards. All features of the standards matter—from the practices to surrounding text including the particular wording of the individual content standards. Guidance for priorities is given at the cluster level as a way of thinking about the content with the necessary specificity yet without going so far into detail as to comprise and coherence of the standards (grain size).

Depth Opportunities

Each cluster, at a grade level, and, each domain at the high school, identifies five or fewer standards for in-depth instruction called Depth Opportunities (Zimba, 2011). Depth Opportunities (DO) is a qualitative recommendation about allocating time and effort within the highest priority clusters --the **Focus** level. Examining the Depth Opportunities by standard reflects that some are beginnings, some are critical moments or some are endings in the progressions. The DO's provide a prioritization for handling the uneven grain size of the content standards. Most of the DO's are not small content elements, but, rather focus on a big important idea that students need to develop.
DO's can be likened to the Priorities in that they are meant to have relevance for instruction, assessment and professional development. In planning instruction related to DO's, teachers need to intensify the mode of engagement by emphasizing: **tight focus**, **rigorous reasoning and discussion** and **extended class time devoted to practice and reflection** and have **high expectation for mastery**. (See Table 6 Appendix, Depth of Knowledge DOK)

In this document, Depth Opportunities are highlighted within a box in the Standards section.

In this document, Depth Opportunities are highlighted pink in the Standards section. For example:

5.NBT.6 *Find whole number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models.*

Depth Opportunities can provide guidance for examining materials for purchase, assist in professional dialogue of how best to develop the DO's in instruction and create opportunities for teachers to develop high quality methods of formative assessment.

Standards for Mathematical Practice in Grade 5

The Common Core State Standards for Mathematical Practice are practices expected to be integrated into every mathematics lesson for all students Grades K-12. Below are a few examples of how these Practices may be integrated into tasks that Grade 5 students complete.

Practice	Explanation and Example
1) Make sense of problems and persevere in solving them.	Mathematically proficient students in Grade 5 solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. Fifth graders may consider different representations of the problem and different solution pathways, both their own and those of other students, in order to identify and analyze correspondences among approaches. When they find that their solution pathway does not make sense, they look for another pathway that does. They check their thinking by asking themselves, "What is the most efficient way to solve the problem?", "Does this make sense?", and "Can I solve the problem in a different way?."
2) Reason abstractly and quantitatively.	Mathematically proficient students in Grade 5 recognize that a number represents a specific quantity. They extend this understanding from whole numbers to their work with fractions and decimals. This involves two processes- decontexualizing and contextualizing. Grade 5 students decontextualize by taking a real-world problem and writing and solving equations based on the word problem. For example, consider the task, "There are $2\frac{2}{3}$ of a yard of rope in the shed. If a total of $4\frac{1}{6}$ yard is needed for a project, how much more rope is needed?" Students decontextualize the problem by writing the equation $4\frac{1}{6} - 2\frac{2}{3} = ?$ and then solving it. Further, students contextualize the problem after they find the answer, by reasoning that 1 3/6 or 1 ½ yards of rope is the amount needed. Further, Grade 4 students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.
3) Construct viable arguments and critique the reasoning of others.	Mathematically proficient students in Grade 5 construct arguments using representations, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking through either discussions or written responses. In Grade 5, students return to their conjectures and arguments about whole numbers to determine whether they apply to fractions and decimals. For example, they might make an argument based on an area representation of multiplication to show that the distributive property applies to problems involving fractions.

4) Model with mathematics.	Mathematically proficient students in Grade 5 experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems. Example, when students encounter situations such as sharing a pan of brownies among 6 people, they might first show how to divide the brownies into 6 equal pieces using a picture of a rectangle. The rectangle divided into 6 equal pieces is a model of the essential mathematics elements of the situation. When the students write the name of each piece in relation to the whole pan as 1/6, they are now modeling the situation with mathematical notation.
5) Use appropriate tools strategically.	Mathematically proficient students in Grade 5 consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data. *Estimation* is also seen as a tool. For example, in order to solve 3/5 – 1/2, a 5^{th} grader might recognize that knowledge of equivalents of 1/2 is an appropriate tool: since 1/2 is equivalent to 2 ½ fifths, the result is ½ of a fifth or 1/10. This practices is also related to looking for structure (SMP 7), which often results in building mathematical tools that can then be used to solve problems.
6) Attend to precision.	Mathematically proficient students in Grade 5 continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.
7) Look for and make use of structure.	Mathematically proficient students in Grade 5 look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation. For example, when 5^{th} graders calculate 16 X 9, they might apply the structure of place value and the distributive property to find the product: 16 X 9 = (10 + 6) x 9 = (10 X 9) + (6 x 9).
8) Look for and express regularity in repeated reasoning.	Mathematically proficient students in Grade 5 use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations. For example, 5^{th} graders might notice a pattern in the change to the product when a factor is increased by 1: 5 X 7 = 35 and 5 X 8 = 40---the product changes by 5; 9 X 4 = 36 and 10 X 4 – 40—the product changes by 4. Fifth graders might then express this regularity by saying something like, *"When you change one factor by 1, the product increases by the other factor."* As students practice articulating their observations, they learn to communicate with greater precisions (SMP 6). As they explain why these generalizations must be true, they construct, critique, and compare arguments (SMP 3).

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
1. **Make sense of problems and persevere in solving them.** • Interpret and make meaning of the problem looking for starting points. Analyze what is given to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Can monitor their progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Can understand various approaches to solutions. • Continually ask themselves; "Does this make sense?"	• How would you describe the problem in your own words? • How would you describe what you are trying to find? • What do you notice about? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. • What might you change? • Talk me through the steps you've used to this point. • What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you organize, represent, and show?
2. **Reason abstractly and quantitatively.** • Make sense of quantities and their relationships. • Are able to decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and are flexible in the use of operations and their properties. • Create a logical representation of the problem. • Attends to the meaning of quantities, not just how to compute them.	• What do the numbers used in the problem represent? • What is the relationship of the quantities? • How is ______related to ______? • What is the relationship between _______and ______? • What does _______mean to you? (e.g. symbol, quantity, diagram) • What properties might we use to find a solution? • How did you decide in this task that you needed to use? • Could we have used another operation or property to solve this task? Why or why not?
3. **Construct viable arguments and critique the reasoning of others.** • Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve/revise the argument. • Compare two arguments and determine correct or flawed logic.	• What mathematical evidence would support your solution? How can we be sure that _________? / How could you prove that._____? Will it still work if._____? • What were you considering when._______? • How did you decide to try that strategy? • How did you test whether your approach worked? • How did you decide what the problem was asking you to find? (What was unknown?) • Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? • What is the same and what is different about.______? • How could you demonstrate a counter-example?
4. **Model with mathematics.** • Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply the math they know to solve problems in everyday life. • Are able to simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving or revising the model. • Ask themselves, "How can I represent this mathematically?"	• What number model could you construct to represent the problem? • What are some ways to represent the quantities? • What's an equation or expression that matches the diagram, number line, chart, table? • Where did you see one of the quantities in the task in your equation or expression? • Would it help to create a diagram, graph, table? • What are some ways to visually represent? • What formula might apply in this situation?

<table>
<tr><th>Summary of Standards for Mathematical Practice</th><th>Questions to Develop Mathematical Thinking</th></tr>
<tr><td>5. Use appropriate tools strategically.
Use available tools recognizing the strengths and limitations of each.
Use estimation and other mathematical knowledge to detect possible errors.
Identify relevant external mathematical resources to pose and solve problems.
Use technological tools to deepen their understanding of mathematics.</td><td>What mathematical tools could we use to visualize and represent the situation?
What information do you have?
What do you know that is not stated in the problem?
What approach are you considering trying first?
What estimate did you make for the solution?
In this situation would it be helpful to use: a graph, number line, ruler, diagram, calculator, manipulative?
Why was it helpful to use.______?
What can using a _____ show us, that ____may not?
In what situations might it be more informative or helpful to use.______?</td></tr>
<tr><td>6. Attend to precision.
Communicate precisely with others and try to use clear mathematical language when discussing their reasoning.
Understand meanings of symbols used in mathematics and can label quantities appropriately.
Express numerical answers with a degree of precision appropriate for the problem context.
Calculate efficiently and accurately.</td><td>What mathematical terms apply in this situation?
How did you know your solution was reasonable?
Explain how you might show that your solution answers the problem.
Is there a more efficient strategy?
How are you showing the meaning of the quantities?
What symbols or mathematical notations are important in this problem?
What mathematical language, definitions, properties can you use to explain.____?
How could you test your solution to see if it answers the problem?</td></tr>
<tr><td>7. Look for and make use of structure.
Apply general mathematical rules to specific situations.
Look for the overall structure and patterns in mathematics.
See complicated things as single objects or as being composed of several objects.</td><td>What observations do you make about._____?
What do you notice when._____?
What parts of the problem might you eliminate, simplify?
What patterns do you find in._____?
How do you know if something is a pattern?
What ideas that we have learned before were useful in solving this problem?
What are some other problems that are similar to this one?
How does this relate to.________?
In what ways does this problem connect to other mathematical concepts?</td></tr>
<tr><td>8. Look for and express regularity in repeated reasoning.
See repeated calculations and look for generalizations and shortcuts.
See the overall process of the problem and still attend to the details.
Understand the broader application of patterns and see the structure in similar situations.
Continually evaluate the reasonableness of their intermediate results.</td><td>Will the same strategy work in other situations?
Is this always true, sometimes true or never true?
How would we prove that._________?
What do you notice about._________?
What is happening in this situation?
What would happen if.________?
What Is there a mathematical rule for.______?
What predictions or generalizations can this pattern support?
What mathematical consistencies do you notice?</td></tr>
</table>

Critical Areas for Mathematics in 5th Grade

1. Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)
   (5.NF.1; 5.NF.2; 5.NF.3; 5.NF.4; 5.NF.6; 5.NF.7)

2. Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.
   (5.NBT.1; 5.NBT.2; 5.NBT.3; 5.NBT.4; 5.NBT.6; 5.NBT.7)

3. Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.
   (5.MD.3; 5.MD.4; 5.MD.5)

Dynamic Learning Maps (DLM) and Essential Elements

The Dynamic Learning Maps and Essential Elements are knowledge and skills linked to the grade-level expectations identified in the Common Core State Standards. The purpose of the Dynamic Learning Maps Essential Elements is to build a bridge from the content in the Common Core State Standards to academic expectations for students with the most significant cognitive disabilities.

For more information please visit the Dynamic Learning Maps and Essential Elements website.

Grade 5 Content Standards Overview

Operations and Algebraic Thinking (OA)

- Write and interpret numerical expressions.
  OA.1 OA.2
- Analyze patterns and relationships.
  OA.3

Number and Operations in Base Ten (NBT)

- Understand the place value system.
  NBT.1 NBT.2 NBT.3 NBT.4
- Perform operations with multi-digit whole numbers and with decimals to hundredths.
  NBT.5 NBT.6 NBT.7

Number and Operations—Fractions (NF)

- Using equivalent fractions as a strategy to add and subtract fractions.
  NF.1 NF.2
- Apply and extend previous understandings of multiplication and division to and divide fractions.
  NF.3 NF.4 NF.5 NF.6 NF.7

Measurement and Data (MD)

- Converts like measurement units within a given measurement system.
  MD.1
- Represent and interpret data.
  MD.2
- Geometric measurement: understand concepts of volume and related volume to multiplication and to addition.
  MD.3 MD.4 MD.5

Geometry (GE)

- Graph points on the coordinate plane to solve real world and mathematical problems.
  G.1 G.2
- Classify two-dimensional figures into categories based on their properties.
  G.3 G.4

Major Supporting Additional **Depth Opportunities(DO)**

Domain: Operations and Algebraic Thinking (OA)

***Cluster:* Writes and interpret numerical expressions.**

Standard: Grade 5. OA.1

Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 - Make sense of problems and persevere in solving them.
- ✓ MP.5 - Use appropriate tools strategically.
- ✓ MP.8 - Look for and express regularity in repeated reasoning.

Connections:

This cluster is connected to:

- Grade 5 Critical Area of Focus #2, extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.
- *See Also: Evaluating numerical Expressions with whole-number exponents (6.OA.1)

Explanation and Examples: 5.OA.1

This standard builds on the expectations of third grade where students are expected to start learning the conventional order. Students need experiences with multiple expressions that use grouping symbols throughout the year to develop understanding of when and how to use parentheses, brackets, and braces. First, students use these symbols with whole numbers. Then the symbols can be used as students add, subtract, multiply and divide decimals and fractions.

Examples:

Problem	Answer
$(26 + 18) \div 4$	11
$\{[2 \times (3 + 5)] - 9\} + [5 \times (23 - 18)]$	32
$12 - (0.4 \times 2)$	11.2
$(2 + 3) \times (1.5 - 0.5)$	5
$6 - \left(\frac{1}{2} + \frac{1}{3}\right)$	$5\frac{1}{6}$
$\left\{80 \div \left[2 \times \left(3\frac{1}{2} + 1\frac{1}{2}\right)\right]\right\} + 100$	108

To further develop students' understanding of grouping symbols and facility with operations, students place grouping symbols in equations to make the equations true or they compare expressions that are grouped differently.

Examples:

$15 - 7 - 2 = 10 \rightarrow 15 - (7 - 2) = 10$
$3 \times 125 \div 25 + 7 = 22 \rightarrow [3 \times (125 \div 25)] + 7 = 22$
$24 \div 12 \div 6 \div 2 = 2 \times 9 + 3 \div \frac{1}{2} \rightarrow 24 \div [(12 \div 6) \div 2] = (2 \times 9) + \left(3 \div \frac{1}{2}\right)$
Compare $3 \times 2 + 5$ and $3 \times (2 + 5)$
Compare $15 - 6 + 7$ and $15 - (6 + 7)$

Instructional Strategies: 5.OA.1-2

Students should be given ample opportunities to explore numerical expressions with mixed operations. This is the foundation for evaluating numerical and algebraic expressions that will include whole-number exponents in Grade 6.

There are conventions (rules) determined by mathematicians that must be learned with no conceptual basis. For example, multiplication and division are always done before addition and subtraction.

Begin with expressions that have two operations without any grouping symbols (multiplication or division combined with addition or subtraction) before introducing expressions with multiple operations. Using the same digits, with the operations in a different order, have students evaluate the expressions and discuss why the value of the expression is different. For example, have students evaluate $5 \times 3 + 6$ and $5 + 3 \times 6$.

Discuss the rules that must be followed. Have students insert parentheses around the multiplication or division part in an expression. A discussion should focus on the similarities and differences in the problems and the results. This leads to students being able to solve problem situations which require that they know the order in which operations should take place.

After students have evaluated expressions without grouping symbols, present problems with one grouping symbol, beginning with parentheses, then in combination with brackets and/or braces. Have students write numerical expressions in words without calculating the value. This is the foundation for writing algebraic expressions. Then, have students write numerical expressions from phrases without calculating them.

Resources/Tools

5.OA Picturing Factors in Different Orders
5.MD,OA You Can Multiply Three Numbers in Any Order
5.OA Watch Out for Parentheses 1
5.OA Bowling for Numbers
5.OA Using Operations and Parentheses

Common Misconceptions:

Students may believe the order in which a problem with mixed operations is written is the order to solve the problem. The use of mnemonic phase "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally" to remember the order of operations (*Parentheses, Exponents, Multiplication, Division, Addition, Subtraction)* can also mislead students to always perform multiplication before division and addition before subtraction. To correct this thinking, students need to understand that addition and subtraction are inverse operations and multiplication and division are inverse operations, as in they have the same "impact". At this level, students need opportunities to explore the "impact" of the various operations on numbers and solve equations starting with the operation of greatest "impact".

Example:

$3 + 2 = 5, 5 - 2 = 3$ (generalize subtraction "undoes" addition – inverse operation)
$3 \times 2 = 6, 6 \div 2 = 3$ (generalize division "undoes" multiplication – inverse operation and multiplication and division have a greater "impact" on a number than addition and subtraction)
$3^2 = 9$ (generalize, exponents have a greater "impact" on a number)

Allow students to use calculators to determine the value of the expression and then discuss the order the calculator used to evaluate the expression. Do this with four-function and scientific calculators.

Students need lots of experience with writing multiplication in different ways. Multiplication can be indicated with a raised dot, such as $4 \cdot 5$, with a raised cross symbol, such as 4×5, or with parentheses, such as $4(5)\ or\ (4)(5)$. Note that the raised cross symbol is *not the same as* the letter "x", and so care should be taken when writing or typing it. Students need to be exposed to all three notations and should be challenged to understand that all are useful. In instruction, teachers are encouraged to use a notation and stay consistent. Students also need help and practice remembering the convention that we write *a* rather than $1 \cdot a$ or $1a$, especially in expressions such as $a + 3a$.

Domain: Operations and Algebraic Thinking (OA)

Cluster: *Write and interpret numerical expressions.*

Standard: Grade 5. OA.2

Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as* $2 \times (8 + 7)$*. Recognize that* $3 \times (18932 + 921)$ *is three times as large as* $18932 + 921$*, without having to calculate the indicated sum or product.*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 **-** Make sense of problems and persevere in solving them.
- ✓ MP.2 - Reason abstractly and quantitatively.
- ✓ MP.7 - Look for and make use of structure.
- ✓ MP.8 - Look for and express regularity in repeated reasoning.

Connections: **See Grade 5.OA.1**

Explanation and Examples:

This standard refers to expressions. Expressions are a series of numbers and symbols $(+, -, \times, \div)$ without an equals sign. Equations result when two expressions are set equal to each other $(2 + 3 = 4 + 1)$.

Example:

$4(5 + 3)$ is an expression.

When we compute $4(5 + 3)$ we are evaluating the expression. The expression equals 32. $4(5 + 3) = 32$ is an equation.

This standard also calls for students to verbally describe the relationship between expressions without actually calculating them. This standard calls for students to apply their reasoning of the four operations as well as place value while describing the relationship between numbers. The standard does not include the use of variables, only numbers and signs for operations.

Example:

Teacher: Write an expression for the steps "double five and then add 26."

Student: $(2 \times 5) + 26$

Teacher: Describe how the expression $5(10 \times 10)$ relates to (10×10)

Student: The expression $5(10 \times 10)$ is 5 times larger than the expression (10×10) since I know that $5(10 \times 10)$ means that I have 5 groups of (10×10). Students use their understanding of operations and grouping symbols to write expressions and interpret the meaning of a numerical expression.

Other Examples:

- Students write an expression for calculations given in words such as "divide 144 by 12 and then subtract $\frac{7}{8}$." They write $(144 \div 12) - \frac{7}{8}$.
- Students recognize that $0.5 \times (300 \div 15)$ is $\frac{1}{2}$ of $(300 \div 15)$ without calculating the quotient.

Instructional Strategies: **See Grade 5.OA.1**

Resources/Tools

For detailed information, see Learning Progression Operations and Algebraic Thinking:
http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k5_2011_05_302.pdf

5.OA Words to Expressions 1
5.OA Video Game Scores
5.OA Comparing Products
5.OA Seeing is Believing

Common Misconceptions: See 5.OA. 1

Domain: Operations and Algebraic Thinking (OA)

Cluster: *Analyze patterns and relationships.*

Standard: Grade 5. OA.3

Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.7 Look for and make use of structure.

Connections:

- This Cluster goes beyond the Grade 5 Critical Area of Focus to address the concepts of modeling numerical relationships with the coordinate plane.
- Generate and analyze patterns (4.OA.3).
- Graphing points in the first quadrant of a coordinate plane (5.G.1-2).

Explanation and Examples:

This standard extends the work from Fourth Grade, where students generate numerical patterns when they are given one rule. In Fifth Grade, students are given two rules and generate two numerical patterns. The graphs that are created should be line graphs to represent the pattern. This is a linear function which is why we get the straight lines. The Days are the independent variable, Fish are the dependent variables, and the constant rate is what the rule identifies in the table.

Example:

Make a chart (table) to represent the number of fish that Joe and Melisa catch.

DAYS	Melisa's Total Number of Fish	Joe's Total Number of Fish
0	0	0
1	2	4
2	4	8
3	6	12
4	8	16
5	10	20

Example:

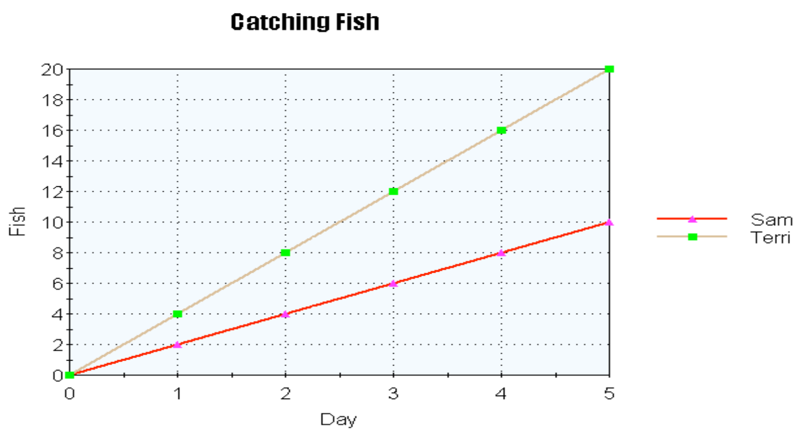
Since Joe catches 4 fish each day, and Melisa catches 2 fish, the amount of Joe's fish is always greater. Joe's fish is also always twice as much as Melisa's fish. Today, both Melisa and Joe have no fish. They both go fishing each day. Melisa catches 2 fish each day. Joe catches 4 fish each day. How many fish do they have after each of the five days? Make a graph of the number of fish. Plot the points on a coordinate plane and make a line graph, and then interpret the graph

Describe the pattern:

Student: My graph shows that Joe always has more fish than Melisa. Joe's fish increases at a higher rate since he catches 4 fish every day. Melisa only catches 2 fish each day so her number of fish increases at a smaller rate than Joe's.

Items to Note:

- It is important to note that the lines become increasingly further apart as well as show a relationship between corresponding terms.
- Note also that the two lines will never intersect; there will not be a day in which Melisa and Joe have the same total of fish,
- This Graph also explain the relationship between the number of days that has passed and the number of fish each person has (2n or 4n, n being the number of days).



Example:

Use the rule "add 3" to write a sequence of numbers. Starting with a zero, students write 0, 3, 6, 9, 12 . . .

Use the rule "add 6" to write a sequence of numbers. Starting with zero, students write 0, 6, 12, 18, 24 . . .

After comparing these two sequences, the students notice that each term in the second sequence is twice the corresponding terms of the first sequence. One way they justify this is by describing the patterns of the terms. Their justification may include some mathematical notation (See example below).

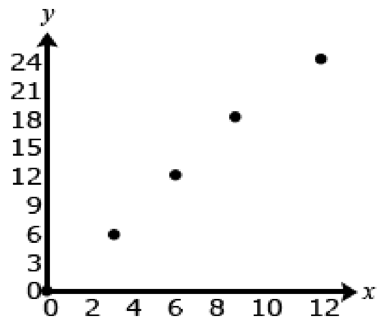
A student may explain that both sequences start with zero and to generate each term of the second sequence he/she added 6, which is twice as much as was added to produce the terms in the first sequence. Students may also use the distributive property to describe the relationship between the two numerical patterns by reasoning that $6 + 6 + 6 = 2(3 + 3 + 3)$.

$$0, \quad {}^{+3}3, \quad {}^{+3}6, \quad {}^{+3}9, \quad {}^{+3}12$$
$$0, \quad {}^{+6}6, \quad {}^{+6}12, \quad {}^{+6}18, \quad {}^{+6}24$$

Once students can describe that the second sequence of numbers is twice the corresponding terms of the first sequence, the terms can be written in ordered pairs and then graphed on a coordinate grid. They should recognize that each point on the graph represents two quantities in which the second quantity is twice the first quantity

Ordered Pairs

$$(0,0)\ (3,6)\ (6,12)\ (9,18)\ (12,24)$$



Instructional Strategies:
Students should have experienced generating and analyzing numerical patterns using a given rule in Grade 4.
Given two rules with an apparent relationship, students should be able to identify the relationship between the resulting sequences of the terms in one sequence to the corresponding terms in the other sequence. For example, starting with 0, multiply by 4 and starting with 0, multiply by 8 which generates each sequence of numbers (0, 4, 8, 12, 16, …) and (0, 8, 16, 24, 32,…). Students should see that the terms in the second sequence are double the terms in the first sequence, or that the terms in the first sequence are half the terms in the second sequence.

Have students form ordered pairs and graph them on a coordinate plane. Patterns can be also observed from the graphs.

Graphing ordered pairs on a coordinate plane (as show above) is introduced to students in the Geometry domain where students solve real-world and mathematical problems. For the purpose of this cluster, use only the first quadrant of the coordinate plane, (which contains positive numbers) only. Provide coordinate grids for the students, but also have them make coordinate grids. In Grade 6, students will position pairs of integers on a coordinate plane.

The graph of both sequences of numbers is a visual representation that will show the relationship between the two sequences of numbers. Encourage students to represent the sequences in T-Charts so they can see a connection between the graph and the sequences.

Resources/Tools
See engageNY Module 6:
https://www.engageny.org/resource/grade-5-mathematics-module-6

Common Misconceptions:
Students reverse the points when plotting them on a coordinate plane. They count up first on the y-axis and then count over on the x-axis. The location of every point in the plane has a specific place. Have students plot points where the numbers are reversed such as (4, 5) and (5, 4). Begin with students providing a verbal description of how to plot each point. Then, have them follow the verbal description and plot each point.

Domain: Number and Operations Base Ten (NBT)

Cluster: *Understand the place value system.*

Standard: Grade 5.NBT.1

Recognize that in a multi-digit number, a digit in the one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to the left.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: (5.NBT.1-4)

This cluster is connected to:

- Grade 5 Critical Area of Focus #2, Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.
- Understand decimal notation for fractions, and compare decimal fractions (Grade 4 NF 7).
- Students need to have a firm grasp of place value for future work with computing with numbers, exponents and scientific notation.

Explanation and Examples:

This standard calls for students to reason about the magnitude of numbers. Students should work with the idea that the tens place is ten times as much as the ones place, and the ones place is $\frac{1}{10}th$ the size of the tens place.

In fourth grade, students examined the relationships of the digits in numbers for whole numbers only. This standard extends this understanding to the relationship of decimal fractions. Students use base ten blocks, pictures of base ten blocks, and interactive images of base ten blocks to manipulate and investigate the place value relationships. They use their understanding of unit fractions to compare decimal places and fractional language to describe those comparisons.

Before considering the relationship of decimal fractions, students express their understanding that in multi-digit whole numbers, a digit in one place represents 10 times what it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.

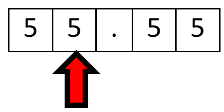
Example:

The 2 in the number 542 is different from the value of the 2 in 324. The 2 in 542 represents 2 ones or 2, while the 2 in 324 represents 2 tens or 20. Since the 2 in 324 is one place to the left of the 2 in 542 the value of the 2 is 10 times greater. Meanwhile, the 4 in 542 represents 4 tens or 40 and the 4 in 324 represents 4 ones or 4. Since the 4 in 324 is one place to the right of the 4 in 542 the value of the 4 in the number 324 is $\frac{1}{10}th$ of its value in the number 542.

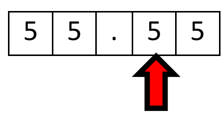
A student thinks, "I know that in the number 5555, the 5 in the tens place (5555) represents 50 and the 5 in the hundreds place (5555) represents 500. So a 5 in the hundreds place is ten times as much as a 5 in the tens place or a 5 in the tens place is $\frac{1}{10}$ of the value of a 5 in the hundreds place".

Based on the base-10 number system digits to the left are times as great as digits to the right; likewise, digits to the right are $\frac{1}{10}th$ of digits to the left. For example, the 8 in 845 has a value of 800 which is ten times as much as the 8 in the number 782. In the same spirit, the 8 in 782 is $\frac{1}{10}th$ the value of the 8 in 845.

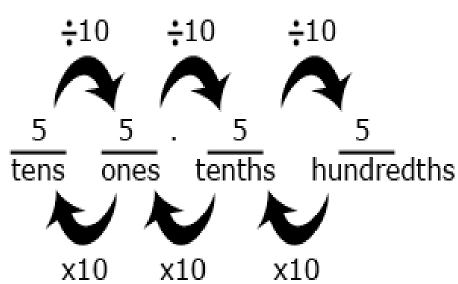
Example:
To extend this understanding of place value to their work with decimals, students use a model of one unit; they cut it into 10 equal pieces, shade in, or describe 1/10 of that model using fractional language ("This is 1 out of 10 equal parts. So it is $\frac{1}{10}$". I can write this using $\frac{1}{10}$ or 0.1"). They repeat the process by finding $\frac{1}{10}$ of a $\frac{1}{10}$ (e.g., dividing $\frac{1}{10}$ into 10 equal parts to arrive at $\frac{1}{100}$ or 0.01) and can explain their reasoning, "0.01 is $\frac{1}{10}$ of $\frac{1}{10}$ thus is $\frac{1}{100}$ of the whole unit."
In the number 55.55, each digit is 5, but the value of the digits is different because of the placement.



The 5 that the arrow points to is $\frac{1}{10}$ of the 5 to the left and 10 times the 5 to the right. The 5 in the ones place is $\frac{1}{10}$ of 50 and 10 times five tenths.



The 5 that the arrow points to is $\frac{1}{10}$ of the 5 to the left and 10 times the 5 to the right. The 5 in the tenths place is 10 times five hundredths.



Instructional Strategies: (5.NBT.1-4)

In Grade 5, the concept of place value is extended to include decimal values to thousandths. The strategies for Grades 3 and 4 should be drawn upon and extended for whole numbers and decimal numbers. For example, students need to continue to represent, write and state the value of numbers including decimal numbers. For students who are not able to read, write and represent multi-digit numbers, working with decimals will be challenging.

Money is a good medium to compare decimals. Present contextual situations that require the comparison of the cost of two items to determine the lower or higher priced item. Students should also be able to identify how many pennies, dimes, dollars and ten dollars, etc., are in a given value. Help students make connections between the number of each type of coin and the value of each coin, and the expanded form of the number. Build on the understanding that it always takes ten of the number to the right to make the number to the left.

Number cards, number cubes, spinners and other manipulatives can be used to generate decimal numbers. For example, have students roll three number cubes, then create the largest and small number to the thousandths place. Ask students to represent the number with numerals and words.

Instructional Resources/Tools:

For detailed information see: Learning Progressions for Numbers and Operations in Base Ten

5.NBT Are these equivalent to 9.52?
5.NBT Kipton's Scale
5.NBT Tenths and Hundredths
5.NBT Which number is it?
5.NBT Millions and Billions of People

Common Misconceptions: (5.NBT.1-4)

A common misconception that students have when trying to extend their understanding of whole number place value to decimal place value is that as you move to the left of the decimal point, the number increases in value. Reinforcing the **concept of powers of ten** is essential for addressing this issue.

A second misconception that is directly related to comparing whole numbers is the idea that the longer the number the greater the number. With whole numbers, a 5-digit number is always greater than a 1-, 2-, 3-, or 4-digit number. However, with decimals a number with one decimal place may be greater than a number with two or three decimal places. For example, 0.5 is greater than 0.12, 0.009 or 0.499.

Domain: Number and Operations in Base Ten (NBT)

***Cluster:** Understand the place value system.*

Standard: Grade 5.NBT.2

Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Standards for Mathematical Practice (MP) to be emphasized:

- ✓ MP.2. Reason abstractly and quantitatively.
- ✓ MP.6. Attend to precision.
- ✓ MP.7. Look for and make use of structure.

Connections: See Grade 5.NBT.1

Explanation and Examples:

This standard includes multiplying by multiples of 10 and powers of 10, including 102 which is $10 \times 10 = 100$, and 103 which is $10 \times 10 \times 10 = 1{,}000$. Students should have experiences working with connecting the pattern of the number of zeros in the product when you multiply by powers of 10.

Example:

$2.5 \times 10^3 = 2.5(10 \times 10 \times 10) = 2.5 \times 1{,}000 = 2{,}500$. Students should reason that the exponent above the 10 indicates how many places the decimal point is moving (not just that the decimal point is moving but that you are multiplying or making the number 10 times greater three times) when you multiply by a power of 10. Since we are multiplying by a power of 10 the decimal point moves to the right.

$$350 \div 10^3 = 350 \div 1{,}000 = 0.350 = 0.35$$
$$350 \div 10 = 35, 35 \div 10 = 3.5$$
$$3.5 \div 10 = 0.35, or\ 350 \times \frac{1}{10}, 35 \times \frac{1}{10}, 3.5 \times \frac{1}{10},$$

This will relate well to subsequent work with operating with fractions. This example shows that when we divide by powers of 10, the exponent above the 10 indicates how many places the decimal point is moving (how many times we are dividing by 10 , the number becomes ten times smaller). Since we are dividing by powers of 10, the decimal point moves to the left.

Provides students with many opportunities to explore this concept and come to this <u>understanding</u>; this should <u>not just be taught procedurally</u>.

Students might write:	Students might think and/or say:
• $36 \times 10 = 36 \times 10^1 = 360$ • $36 \times 10 \times 10 = 36 \times 10^2 = 3{,}600$ • $36 \times 10 \times 10 \times 10 = 36 \times 10^3 = 36{,}000$ • $36 \times 10 \times 10 \times 10 \times 10 = 36 \times 10^4 = 360{,}000$	**I noticed that every time, I multiplied by 10 I added a zero to the end of the number. That makes sense because each digit's value became 10 times larger. To make a digit 10 times larger, I have to move it one place value to the left.* **When I multiplied 36 by 10, the 30 became 300. The 6 became 60 or the 36 became 360. So I had to add a zero at the end to have the 3 represent 3 one-hundreds (instead of 3 tens) and the 6 represents 6 tens (instead of 6 ones).*

Students should be able to use the same type of reasoning as above to **explain why** the following multiplication and division problem by powers of 10 make sense.

- $523 \times 10^3 = 523{,}000$ The place value of 523 is increased by 3 places.
- $5.223 \times 10^2 = 522.3$ The place value of 5.223 is increased by 2 places.
- $52.3 \times 10^1 = 5.23$ The place value of 52.3 is decreased by one place.

Instructional Strategies: See Grade 5.NBT.1

Resources/Tools

5.NBT Marta's Multiplication Error
5.NBT.1 Multiplying Decimals by 10

Common Misconceptions: See Grade 5.NBT.1

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Understand the place value system.*

Standard: Grade 5.NBT.3

Read, write, and compare decimals to thousandths.

a) Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times \left(\frac{1}{10}\right) + 9 \times \left(\frac{1}{100}\right) + 2 \times \left(\frac{1}{1000}\right)$.

b) Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 5.NBT.1

Explanation and Examples: 5.NBT.3a

This standard references expanded form of decimals with fractions included. Students should build on their work from Fourth Grade, where they worked with both decimals and fractions interchangeably. Expanded form is included to build upon work in 5.NBT.2 and deepen students' understanding of place value.

Students build on the understanding they developed in fourth grade to read, write, and compare decimals to thousandths. They connect their prior experiences with using decimal notation for fractions and addition of fractions with denominators of 10 and 100.

They use concrete models and number lines to extend this understanding to decimals to the thousandths. Models may include base ten blocks, place value charts, grids, pictures, drawings, manipulatives, technology-based, etc.

They read decimals using fractional language and write decimals in fractional form, as well as in expanded notation. This investigation leads them to understanding equivalence of decimals (0.8 = 0.80 = 0.800).

5.NBT.3b

Comparing decimals builds on work from fourth grade.

Example:
Some equivalent forms of 0.72 are:

$\frac{72}{100}$	$\frac{7}{10}+\frac{2}{100}$
$7 \times \left(\frac{1}{10}\right) + 2 \times \left(\frac{1}{100}\right)$	$0.70 + 0.02$
$\frac{70}{100}+\frac{2}{100}$	0.720
$7 \times \frac{1}{10} + 2 \times \frac{1}{100} + 0 \times \frac{1}{1000}$	$\frac{720}{1000}$

Students need to understand the size of decimal numbers and relate them to **common benchmarks** such as 0, 0.5 (0.50 and 0.500), and 1. Comparing tenths to tenths, hundredths to hundredths, and thousandths to thousandths is simplified if students use their understanding of fractions to compare decimals.

Example:
Comparing 0.25 and 0.17, a student might think, "25 hundredths is more than 17 hundredths". They may also think that it is 8 hundredths more. They may write this comparison as $0.25 > 0.17$ and recognize that $0.17 < 0.25$ is another way to express this comparison.

Comparing 0.207 to 0.26, a student might think, "Both numbers have 2 tenths, so I need to compare the hundredths. The second number has 6 hundredths and the first number has no hundredths so the second number must be larger.

Another student might think while writing fractions, "I know that 0.207 is 207 thousandths (and may write $\frac{207}{1000}$). 0.26 is 26 hundredths (and may write $\frac{26}{100}$) but I can also think of it as 260 thousandths ($\frac{260}{1000}$). So, 260 thousandths is more than 207 thousandths.

Students build on the understanding they developed in fourth grade to read, write, and compare decimals to thousandths. They connect their prior experiences with using decimal notation for fractions and addition of fractions with denominators of 10 and 100. They use concrete models and number lines to extend this understanding to decimals to the thousandths.

Models may include base ten blocks, place value charts, grids, pictures, drawings, manipulatives, technology-based, etc. They read decimals using fractional language and write decimals in fractional form, as well as in expanded notation as show in the standard 3a. This investigation leads them to understanding equivalence of decimals ($0.8 = 0.80 = 0.800$).

$\frac{72}{100}$	$\frac{7}{10}+\frac{2}{100}$
$7 \times \left(\frac{1}{10}\right) + 2 \times \left(\frac{1}{100}\right)$	$0.70 + 0.02$
$\frac{70}{100}+\frac{2}{100}$	0.720
$7 \times \frac{1}{10} + 2 \times \frac{1}{100} + 0 \times \frac{1}{1000}$	$\frac{720}{1000}$

Instructional Strategies: See Grade 5.NBT.1

Resources/Tools:
See engageNY Module 1

5.NBT Drawing Pictures to Illustrate Decimal Comparisons

Common Misconceptions: See Grade 5.NBT.1

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Understand the place value system.*

Standard: Grade 5.NBT.4

Use place value understanding to round decimals to any place.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 5.NBT.1

Explanation and Examples:

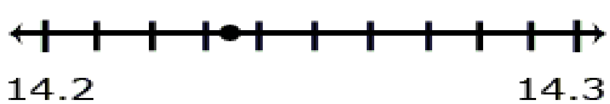
This standard refers to rounding. **Students should go beyond simply applying an algorithm or procedure for rounding**. The expectation is that students have a <u>deep understanding</u> of place value and number sense and can explain and reason about the answers they get when they round.

Students should have numerous experiences using a number line to support their work with rounding. When rounding a decimal to a given place, students may identify the two possible answers, and use their understanding of place value to compare the given number to the possible answers.

Example:

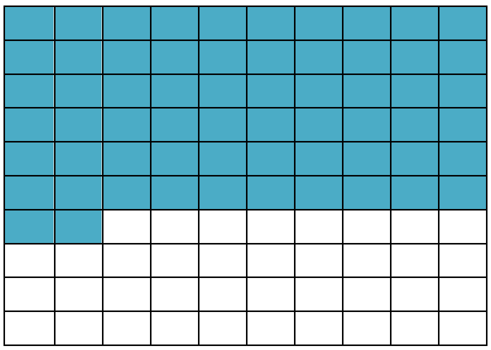
Round 14.235 to the nearest tenth.

Students recognize that the possible answer must be in tenths thus, it is either 14.2 or 14.3. They then identify that 14.235 is closer to 14.2 (14.20) than to 14.3 (14.30).



Students should use **benchmark** numbers to support this work. Benchmarks are convenient numbers for comparing and rounding numbers. 0., 0.5, 1, 1.5 are examples of benchmark numbers.

Example:



Which benchmark number is the best estimate of the shaded amount in the model to the left?

Explain your thinking.

Instructional Strategies: See Grade 5 NBT.1

Instructional Resources/Tools

[NBT Rounding to Tenths and Hundredths]

Common Misconceptions: See Grade 5.NBT.1

Domain: Number and Operations in Base Ten (NBT)

Cluster: *Perform operations with multi-digit whole numbers and with decimals to hundredths.*

Standard: Grade 5.NBT.5

Fluently multiply multi-digit whole numbers using the standard algorithm**.**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: (5.NBT.5-7)

This cluster is connected to:

- Grade 5 Critical Area of Focus #2, Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.
- Use place value understanding and properties of operations to perform multi-digit arithmetic (Grade 4 NBT 5 and 6).

Explanation and Examples:

This standard refers to fluency which means students select and use a variety of methods and tools to compute, including objects, mental computation, estimation, paper and pencil, and calculators.

- They work flexibly with basic number combinations and use visual models, benchmarks, and equivalent forms.
- They are accurate and efficient (use a reasonable amount of steps), and flexible (use strategies such as the distributive property or breaking numbers apart (decomposing and recomposing) also using strategies according to the numbers in the problem, 26×4 may lend itself to $(25 \times 4) + 4$ where as another problem might lend itself to making an equivalent problem $32 \times 4 = 64 \times 2$.

This standard builds upon students' work with multiplying numbers in third and fourth grade. In fourth grade, students developed understanding of multiplication through using various strategies. While the standard algorithm is mentioned, alternative strategies are also appropriate to help students develop conceptual understanding.

The size of the numbers should **NOT** exceed a three-digit factor by a two-digit factor.

In prior grades, students used various strategies to multiply. Students can continue to use these different strategies as long as they are efficient, but must also understand and be able to use the standard algorithm. In applying the standard algorithm, students recognize the importance of place value.

Example:
Find the product of 123×34. When students apply the standard algorithm, they, decompose 34 into $30 + 4$. Then they multiply 123 by 4, the value of the number in the ones place, and then multiply 123 by 30, the value of the 3 in the tens place, and add the two products. The ways in which students are taught to record this method may vary, but ALL should emphasize the place-value nature of the algorithm, For example, one might write

123	
×34	
492 ←	this is the product of 4 and 123
3690 ←	this is the product of 30 and 123
4182 ←	this is the produce of the two partial products

Note that a further decomposition of 123 into 100 + 20 + 30 and recording of the partial products would also be acceptable.

Examples of alternative strategies:
There are 225 dozen cookies in the bakery. How many cookies are there?

Student 1

- 225×12
- *I broke 12 up into 10 and 2.*
- $225 \times 10 = 2{,}250$
- $225 \times 2 = 450$
- $2{,}250 + 450 = 2{,}700$

Student 2

- 225×12
- *I broke up 225 into 200 and 25.*
- $200 \times 12 = 2{,}400$
- *I broke 25 up into* 5×5*, so I had* $5 \times 5 \times 12$ *or* $5 \times 12 \times 5$.
- $5 \times 12 = 60, 60 \times 5 = 300$
- *I then added 2,400 and 300* $2{,}400 + 300 = 2{,}700$

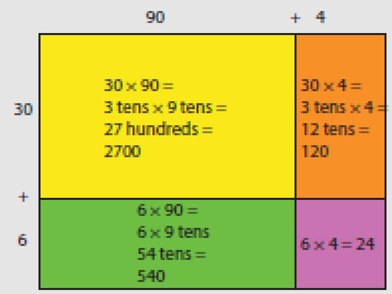
Student 3

- I doubled 225 and cut
- 12 in half to get 450×6.
- I then doubled 450
- again and cut 6 in half
- to get 900×3.
- $900 \times 3 = 2{,}700$

Draw an array model for

$$36 \times 94 = (30 + 6) \times (90 + 4) = (30 + 6) \times 90 + (30 + 6) \times 4 = 30 \times 90 + 6 \times 90 + 30 \times 4 + 6 \times 4$$

Computation of 36×94 connected with an area model



The products of like base-ten units are shown as parts of a rectangular region.

Taken from Progression for the Common Core: K-5, Number and Operations in Base Ten
(Click array to open complete document.)

Instructional Strategies: (5.NBT.5-7)

Because students have used various models and strategies to solve problems involving multiplication with whole numbers, they should be able to transition to using standard algorithms effectively. With guidance from the teacher, they should understand the connection between the standard algorithm and their strategies.
Connections between the algorithm for multiplying multi-digit whole numbers and strategies such as partial products or lattice multiplication are **necessary for students' understanding.**

You can multiply by listing all the partial products. For example:

234	
× 8	
32	Multiply by the ones ($8 \times 4\ ones = 32\ ones$)
240	Multiply by the tens ($8 \times 3\ tens = 24\ tens\ or\ 240$)
1,600	Multiply the hundreds ($8 \times 2\ hundreds = 16\ hundreds\ or\ 1{,}600$)
1,872	Add the partial products

The multiplication can also be done without listing the partial products by multiplying the value of each digit from one factor by the value of each digit from the other factor. Understanding of place value is vital in using the standard algorithm.

In using the standard algorithm for multiplication, when multiplying the ones, 32 ones is 3 tens and 2 ones. The 2 is written in the ones place. When multiplying the tens, the 24 tens is 2 hundreds and 4 tens. But, the 3 tens from the 32 ones need to be added to these 4 tens, for 7 tens. Multiplying the hundreds, the 16 hundreds is 1 thousand and 6 hundreds. But, the 2 hundreds from the 24 tens need to be added to these 6 hundreds, for 8 hundreds.

234
× 8
1,872

As students developed efficient strategies to do whole number operations, they should also develop efficient strategies with decimal operations.

Students should learn to **estimate** decimal computations **before** they compute with pencil and paper. The focus on estimation should be on the meaning of the numbers and the operations, not on how many decimal places are involved.

For example, to estimate the product of 32.84×4.6, the estimate would be more than 120, closer to 150. Students should consider that 32.84 is closer to 30 and 4.6 is closer to 5. The product of 30 and 5 is 150. Therefore, the product of 32.84×4.6 should be close to 150. *(Writing equations horizontally encourages using mental math).*

Have students use estimation to find the product by using exactly the same digits in one of the factors with the decimal point in a different position each time. For example, have students estimate the product of 275×3.8; 27.5×3.8 and 2.75×3.8, and discuss **why** the estimates should or should not be the same.

Instructional Resources/Tools:

5.NBT Elmer's Multiplication Error

Common Misconceptions: (5.NBT.5-7)

Students might compute the sum or difference of decimals by lining up the right-hand digits as they would whole number. For example, in computing the sum of 15.34 + 12.9, students will write the problem in this manner:

$$\begin{array}{r} 15.34 \\ +\ \ 12.9 \\ \hline 16.63 \end{array}$$

To help students add and subtract decimals correctly, have them **first estimate** the sum or difference. Providing students with a decimal-place value chart will enable them to place the digits in the proper place.

Domain: Number and Operations in Base Ten (NBT)

Cluster: Perform operations with multi-digit whole numbers and with decimals to hundredths.

Standard: Grade 5.NBT.6

Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 5.NBT.5

Explanation and Examples:

This standard references various strategies for division. Division problems can include remainders. Even though this standard leads more towards computation, the connection to story contexts is **critical**.

Make sure students are exposed to problems where the divisor is the number of groups and where the divisor is the size of the groups. In fourth grade, students' experiences with division were limited to dividing by one-digit divisors.

This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.

Example:
There are 1,716 students participating in Field Day. They are put into teams of 16 for the competition. How many teams get created? If you have left over students, what do you do with them?

<u>Student 1</u>

1,716 divided by 16

There are 100 16's in 1,716.

$$1{,}716 - 1{,}600 = 116$$

I know there are at least 6 16's.

$$116 - 96 = 20$$

I can take out at least 1 more 16.

$$20 - 16 = 4$$

There were 107 teams with 4 students left over. If we put the extra students on different team, 4 teams will have 17students.

<u>Student 2</u>

1,716 divided by 16

There are 100 16's in 1,716.

Ten groups of 16 is 160.

That's too big.

Half of that is 80, which is5 groups.

I know that 2 groups of 16's is 32.

I have 4 students left over.

$1{,}716 - 1{,}600$	100
$116 - 80$	5
36-32	2
4	

<u>Student 3</u>

$$1{,}716 \div 16 = ?$$

I want to get to 1,716

I know that 100 16's equals 1,600

I know that 5 16's equals 80

$$1{,}600 + 80 = 1{,}680$$

Two more groups of 16's equals 32, which gets us to 1,712.

I am 4 away from 1,716.

So we had $100 + 6 + 1 = 107$ teams.

Those other 4 students can just hang out.

<u>Student 4</u>

How many 16's are in 1,716?

We have an area of 1,716.

I know that one side of my array is 16 units long.

I used 16 as the height.

I am trying to answer the question what is the width of my rectangle if the area is 1,716 and the height is 16.

$$100 + 7 = 107\ R4$$

	100	7
16	$100 \times 16 = 1{,}600$	$7 \times 16 = 112$

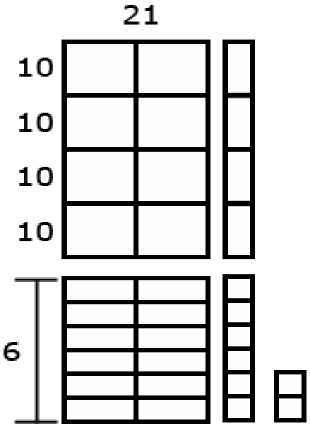
$$1{,}716 - 1{,}600 = 116$$

$$116 - 112 = 4$$

Example:

$968 \div 21 = ?$

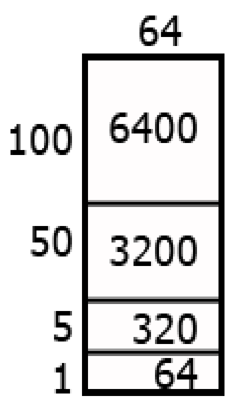
Using base ten models, a student can represent 968 and use the models to make an array with one dimension of 21. The student continues to make the array until no more groups of 21 can be made. Remainders are not part of the array.



Example:

$9{,}984 \div 64 = ?$

An area model for division is shown below. As the student uses the area model, s/he keeps track of how much of the 9984 is left to divide.



$$
\begin{array}{rrl}
64 \,\big) & 9984 & \\
 & -6400 & (100 \times 64) \\
\hline
 & 3584 & \\
 & -3200 & (50 \times 64) \\
\hline
 & 384 & \\
 & -320 & (5 \times 64) \\
\hline
 & 64 & \\
 & -64 & (1 \times 64) \\
\hline
 & 0 &
\end{array}
$$

Instructional Strategies: See Grade 5. NBT. 5

Resources/Tools:

5.MD Minutes and Days

With this activity, you can visually explore the concept of factors by creating rectangular arrays. The length and width of the array are the factors in your number.

Learning Progressions for Numbers and Operations in Base Ten

Common Misconceptions: See Grade 5. NBT. 5

Domain: Number and Operations in Base Ten (NBT)

***Cluster:** Perform operations with multi-digit whole numbers and with decimals to the hundredths.*

Standard: Grade 5.NBT.7

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 5.NBT.5

Explanation and Examples:

This standard builds on the work from fourth grade where students are introduced to decimals and compare them. In fifth grade, students begin adding, subtracting, multiplying and dividing decimals. This work should focus on concrete models and pictorial representations, rather than relying solely on the algorithm. The use of symbolic notations involves having students record the answers to computations ($2.25 \times 3 = 6.75$), but this work should not be done without models or pictures.

This standard includes students' reasoning and explanations of how they use models, pictures, and strategies. This standard requires students to extend the models and strategies they developed for whole numbers in grades 1-4 to decimal values. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers.

Examples:

$$3.6 + 1.7 = ?$$

A student might estimate the sum to be larger than 5 because 3.6 is more than $3\frac{1}{2}$ and 1.7 is more than $1\frac{1}{2}$.

$$5.4 - 0.8 = ?$$

A student might estimate the answer to be a little more than 4.4 because a number less than 1 is being subtracted.

$$6 \times 2.4 = ?$$

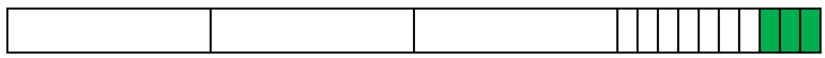
A student might estimate an answer between 12 and 18 since 6×2 is 12 and 6×3 is 18. Another student might give an estimate of a little less than 15 because s/he figures the answer to be very close, but smaller than $6 \times 2\frac{1}{2}$ and think of $2\frac{1}{2}$ groups of 6 as 12 (2 groups of 6) + 3 ($\frac{1}{2}$ of a group of 6).

Students should be able to express that when they add decimals they add tenths to tenths and hundredths to hundredths. So, when they are adding in a vertical format (numbers beneath each other), it is important that they write numbers with the same place value beneath each other. This understanding can be reinforced by connecting addition of decimals to their understanding of addition of fractions. Adding fractions with denominators of 10 and 100 is a standard in fourth grade.

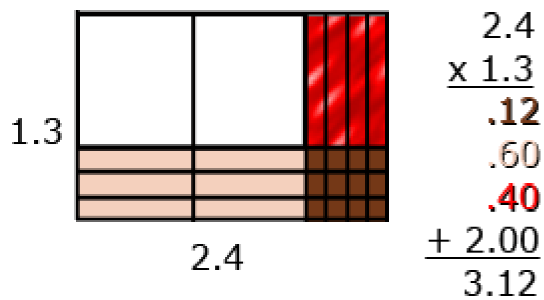
Example:

$$4 - 0.3 = ?$$

3 tenths subtracted from 4 wholes. The wholes must be divided into tenths.



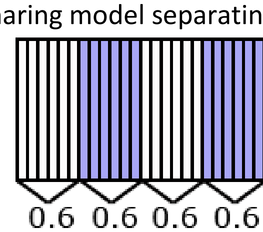
The answer is 3 and $\frac{7}{10}$ or 3.7.



Students should be able to **describe** the partial products displayed by the area model. For example, "$\frac{3}{10}$ times $\frac{4}{10}$ is $\frac{12}{100}$. $\frac{3}{10}$ times 2 is $\frac{6}{10}$ or $\frac{60}{100}$. 1 group of $\frac{4}{10}$ is $\frac{4}{10}$ or $\frac{40}{100}$. 1 group of 2 is 2."

Example of division: finding the number in each group or share

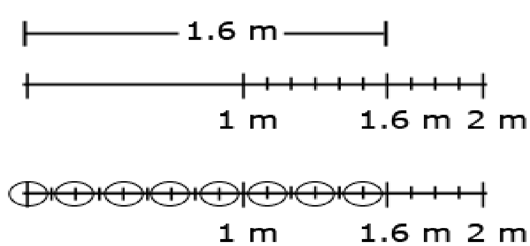
Students should be encouraged to apply a fair sharing model separating decimal values into equal parts such as:



Example of division: find the number of groups

- Joe has 1.6 meters of rope. He has to cut pieces of rope that are 0.2 meters long. How many can he cut?

- To divide to find the number of groups, a student might:
  - Draw a segment to represent 1.6 meters. In doing so, s/he would count in tenths to identify the 6 tenths, and be able identify the number of 2 tenths within the 6 tenths. The student can then extend the idea of counting by tenths to divide the one meter into tenths and determine that there are 5 more groups of 2 tenths.



- Count groups of 2 tenths without the use of models or diagrams. Knowing that 1 can be thought of as $\frac{10}{10}$, a student might think of 1.6 as 16 tenths. Counting 2 tenths, 4 tenths, 6 tenths, . . . 16 tenths, a student can count 8 groups of 2 tenths.
- Use their understanding of multiplication and think, “8 groups of 2 is 16, so 8 groups of $\frac{2}{10}$ is $\frac{16}{10}$ or $1\frac{6}{10}$.”

Instructional Strategies: **See Grade 5.NBT.5**

Resources/Tools:

5.NF What is 23 ÷ 5?

5.NBT The Value of Education

Common Misconceptions: See Grade 5. NBT. 5

Domain: Number and Operations—Fractions (F)

***Cluster:** Uses equivalent fractions as a strategy to add and subtract fractions.*

Standard: Grade 5.NF.1

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{2}{3}+\frac{5}{4}=\frac{8}{12}+\frac{15}{12}=\frac{23}{12}$. (In general, $\frac{a}{b}+\frac{c}{d}=\left(\frac{ad+bc}{bd}\right)$).

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2. Reason abstractly and quantitatively.
- ✓ MP.4. Model with mathematics.
- ✓ MP.7. Look for and make use of structure.

Connections:

This Cluster is connected to:

- Grade 5 Critical Area of Focus #1, Developing fluency with addition and subtraction of fractions and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).
- Develop an understanding of fractions as numbers (Grade 3 NF 3 a–c).

Explanations and Examples:

This standard builds on the work in fourth grade where students add fractions with like denominators. In fifth grade, the example provided in the standard has students find a common denominator by finding the product of both denominators. For $\frac{1}{3}+\frac{1}{6}$, a common denominator is 18, which is the product of 3 and 6. This process should be introduced using visual fraction models (area models, number lines, etc.) to build understanding before moving into the standard algorithm.

Students should apply their understanding of equivalent fractions developed in fourth grade and their ability to rewrite fractions in an equivalent form to find common denominators. They should know that multiplying the denominators will always give a common denominator but may not result in the smallest denominator.

Examples:

1. $\frac{2}{5}+\frac{7}{8}=\frac{16}{40}+\frac{35}{40}=\frac{51}{40}$
2. $3\frac{1}{4}-\frac{1}{6}=3\frac{3}{12}-\frac{2}{12}=3\frac{1}{12}$

Example:

Present students with the problem $\frac{1}{3}+\frac{1}{6}$. Encourage students to use the clock face as a model for solving the problem. Have students share their approaches with the class and demonstrate their thinking using the clock model.

Instructional Strategies: (5.NF1-2)
To add or subtract fractions with unlike denominators, students use their understanding of equivalent fractions to create fractions with the same denominators. Start with problems that require the changing of one of the fractions and progress to changing both fractions. Allow students to add and subtract fractions using different strategies such as number lines, area models, fraction bars or strips. Have students share their strategies and discuss commonalities in them.

Students need to develop the understanding that when adding or subtracting fractions, the fractions must refer to the same whole. Any models used must refer to the same whole. Students may find that a circular model might not be the best model when adding or subtracting fractions.

As with solving word problems with whole number operations, regularly present word problems involving addition or subtraction of fractions. The concept of adding or subtracting fractions with unlike denominators will develop through solving problems.

Mental computations and estimation strategies should be used to determine the reasonableness of answers. Students need to prove or disprove whether an answer provided for a problem is reasonable.

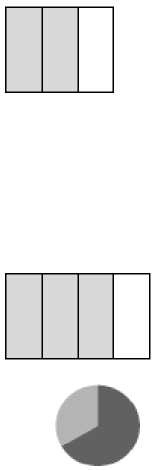
Estimation is about getting useful answers, it is not about getting the right answer. It is important for students to learn which strategy to use for estimation. Students need to think about what might be a close answer and then explain their reasoning.

Instructional Resources/Tools:
For detailed information, see: Learning Progressions for Numbers and Operations - Fractions

5.NF To Multiply or not to multiply?
5.NF.A Measuring Cups
5.NF Egyptian Fractions
5.NF Mixed Numbers with Unlike Denominators
5.NF Finding Common Denominators to Add
5.NF Jog-A-Thon
5.NF Finding Common Denominators to Subtract
5.NF Making S'Mores
5.NF Fractions on a Line Plot

Common Misconceptions:

Students often mix models when adding, subtracting or comparing fractions. Students will use a circle for thirds and a rectangle for fourths when comparing fractions with thirds and fourths. Remind students that the representations need to be from the same whole models with the same shape and size.



These models of fractions are difficult to compare because the size of the whole is not the same for all representations

These models of fractions use the same size rectangle to represent the whole unit and are therefore much easier to compare fractions.

Domain: Number and Operations – Fractions (NF)

***Cluster:** Use equivalent fractions as a strategy to add and subtract fractions.*

Standard: Grade 5.NF.2

Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $\frac{2}{5}+\frac{1}{2}=\frac{3}{7}$, by observing that $\frac{3}{7}<\frac{1}{2}$.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1. Make sense of problems and persevere in solving them.
- ✓ MP.2. Reason abstractly and quantitatively.
- ✓ MP.3. Construct viable arguments and critique the reasoning of others.
- ✓ MP.4. Model with mathematics.
- ✓ MP.5. Use appropriate tools strategically.
- ✓ MP.6. Attend to precision.
- ✓ MP.7. Look for and make use of structure.
- ✓ MP.8. Look for and express regularity in repeated reasoning.

Connections: See Grade 5.NF.1

Explanation and Examples:

This standard refers to number sense, which means students' **understanding of fractions** as numbers that lie between whole numbers on a number line. Number sense in fractions also includes moving between decimals and fractions to find equivalents, also being able to use reasoning such as $\frac{7}{8}$ is greater than $\frac{3}{4}$ because $\frac{7}{8}$ is missing only $\frac{1}{8}$ and $\frac{3}{4}$ is missing $\frac{1}{4}$ so $\frac{7}{8}$ is closer to a whole. Also, students should use **benchmark fractions** to estimate and examine the reasonableness of their answers. Example, $\frac{5}{8}$ is greater than $\frac{6}{10}$ because $\frac{5}{8}$ is $\frac{1}{8}$ larger than $\frac{1}{2}\left(\frac{4}{8}\right)$ and $\frac{6}{10}$ is only $\frac{1}{10}$ larger than $\frac{1}{2}\left(\frac{5}{10}\right)$.

Example:

Your teacher gave you $\frac{1}{7}$ of the bag of candy. She also gave your friend $\frac{1}{3}$ of the bag of candy. If you and your friend combined your candy, what fraction of the bag would you have? Estimate your answer and then calculate. How reasonable was your estimate?

Student 1:

$\frac{1}{7}$ is really close to 0. $\frac{1}{3}$ is larger than $\frac{1}{7}$, but still less than $\frac{1}{2}$. If we put them together we might get close to $\frac{1}{2}$.

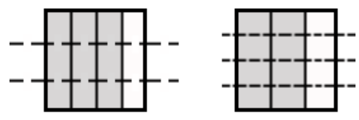
$\frac{1}{7}+\frac{1}{3}=\frac{3}{21}+\frac{7}{21}=\frac{10}{21}$. The fraction does not simplify. I know that 10 is half of 20, so $\frac{10}{21}$ is a little less than $\frac{1}{2}$.

Student 2: $\frac{1}{7}$ is close to $\frac{1}{6}$ but less than $\frac{1}{6}$, and $\frac{1}{3}$ is equivalent to $\frac{2}{6}$, so I have a little less than $\frac{3}{6}$ or $\frac{1}{2}$.

Example:

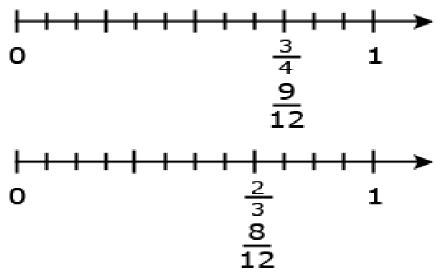
Jerry was making two different types of cookies. One recipe needed $\frac{3}{4}$ cup of sugar and the other needed $\frac{2}{3}$ cup of sugar. How much sugar did he need to make both recipes?

- Mental estimation:
  - A student may say that Jerry needs more than 1 cup of sugar but less than 2 cups. An explanation may compare both fractions to $\frac{1}{2}$ and state that both are larger than $\frac{1}{2}$ so the total must be more than 1. In addition, both fractions are slightly less than 1 so the sum cannot be more than 2.
- Area model

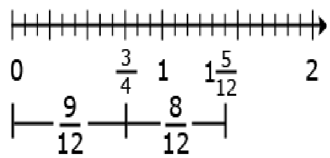


$\frac{3}{4} = \frac{9}{12}$ $\qquad$ $\frac{2}{3} = \frac{8}{12}$ $\qquad$ $\frac{3}{4} + \frac{2}{3} = \frac{17}{12} = \frac{12}{12} + \frac{5}{12} = 1\frac{5}{12}$

- Linear model



Solution:

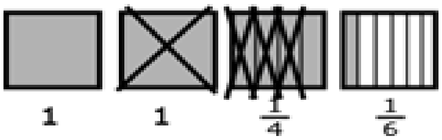


Example: Using a bar diagram

- Melisa had $2\frac{1}{3}$ candy bars. She promised her brother that she would give him $\frac{1}{2}$ of a candy bar. How much will she have left after she gives her brother the amount she promised?
- If Mary ran 3 miles every week for 4 weeks, she would reach her goal for the month. The first day of the first week she ran $1\frac{3}{4}$ miles. How many miles does she still need to run the first week?
  - Using addition to find the answer: $1\frac{3}{4} + n = 3$
  - A student might add $1\frac{1}{4}$ to $1\frac{3}{4}$ to get to 3 miles. Then he or she would add $\frac{1}{6}$ more. Thus $1\frac{1}{4}\ miles + \frac{1}{6}$ of a mile is what Mary needs to run during that week.

Example: Using an area model to subtract

- This model shows 1 ¾ subtracted from 3 1/6 leaving 1 + ¼ + 1/6 which a student can then change to 1 + 3/12 + 2/12 = 1 5/12.

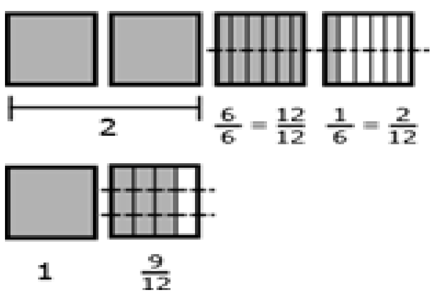


3 $^1/_6$ and 1 ¾ can be expressed with a denominator of 12. Once this is

done a student can complete the problem, 2 14/12 – 1 9/12 = 1 5/12.

- This diagram models a way to show how 3 $^1/_6$ and 1 ¾ can be

expressed with a denominator of 12. Once this is accomplished, a student can complete the problem, 2 14/12 – 1 9/12 = 1 5/12.



Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies for calculations with fractions extend from students' work with whole number operations and can be supported through the use of physical models.

Example:

Elli drank $\frac{3}{5}$ quart of milk and Javier drank $\frac{1}{10}$ of a quart less than Ellie. How much milk did they drink all together?

Solution:

$\frac{3}{5} - \frac{1}{10} = \frac{6}{10} - \frac{1}{10} = \frac{5}{10}$ This is how much milk Javier drank

$\frac{3}{5} + \frac{5}{10} = \frac{6}{10} + \frac{5}{10} = \frac{11}{10}$ Together they drank $1\frac{1}{10}$ quarts of milk

This solution is reasonable because Ellie drank more than $\frac{1}{2}$ quart and Javier drank $\frac{1}{2}$ quart so together they drank slightly more than one quart.

Instructional Strategies: See Grade 5.NF.1

Instructional Tools/Resources:

5.NF Do These Add Up?

5.NF Salad Dressing

Common Misconceptions: See Grade 5.NF.1

Domain: Number and Operations – Fractions (NF)

***Cluster:** Apply and extend previous understandings of multiplication and division to multiply and divide fractions.*

Standard: Grade 5.NF.3

Interpret a fraction as division of the numerator by the denominator $\left(\frac{a}{b} = a \div b\right)$. Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that* $\frac{3}{4}$ *multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size* $\frac{3}{4}$. *If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1. Make sense of problems and persevere in solving them.
- ✓ MP.2. Reason abstractly and quantitatively.
- ✓ MP.3. Construct viable arguments and critique the reasoning of others.
- ✓ MP.4. Model with mathematics.
- ✓ MP.5. Use appropriate tools strategically.
- ✓ MP.7. Look for and make use of structure.

Connections: 5.NF.3-7

This cluster is connected to:

- Grade 5 Critical Area of Focus #1, **Developing fluency with addition and subtraction of fractions and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions)**.
- Foundation for Learning in Grade 6: The Number System, Ratios and Proportional Relationships (Grade 6 RP1).

Explanation and Examples:

This standard calls for students to extend their work of partitioning a number line from third and fourth grade. Students need ample experiences to explore the concept that a fraction is a way to represent the division of two quantities.

Students are expected to demonstrate their understanding using concrete materials, drawing models, and explaining their thinking when working with fractions in multiple contexts. They read $\frac{3}{5}$ as "three fifths" and after many experiences with sharing problems, learn that $\frac{3}{5}$ can also be interpreted as "3 divided by 5."

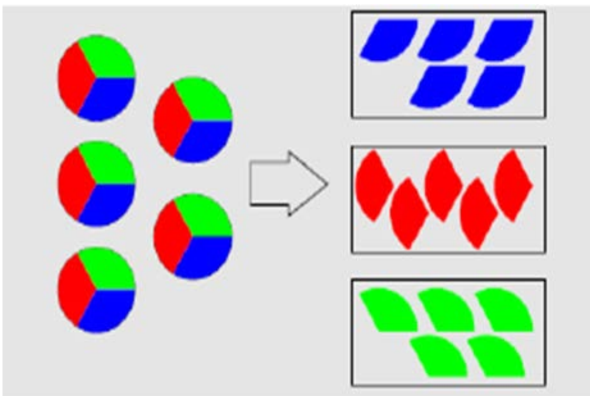
Examples:
Ten team members are sharing 3 boxes of cookies. How much of a box will each student get?

When working this problem a student should recognize that the 3 boxes are being divided into 10 groups, so s/he is seeing the solution to the following equation, $10 \times n = 3$ (10 groups of some amount is 3 boxes) which can also be written as $n = 3 \div 10$. Using models or diagram, they divide each box into 10 groups, resulting in each team member getting $\frac{3}{10}$ of a box.

Two afterschool clubs are having pizza parties. For the Math Club, the teacher will order 3 pizzas for every 5 students. For the student council, the teacher will order 5 pizzas for every 8 students. Since you are in both groups, you need to decide which party to attend. How much pizza would you get at each party? If you want to have the most pizza, which party should you attend?

The six fifth grade classrooms have a total of 27 boxes of pencils. How many boxes will each classroom receive? Students may recognize this as a whole number division problem but should also express this equal sharing problem as $\frac{27}{6}$. They explain that each classroom gets $\frac{27}{6}$ boxes of pencils and can further determine that each classroom get $4\frac{3}{6}$ or $4\frac{1}{2}$ boxes of pencils.

Expect students to demonstrate their understanding using concrete materials, drawing models, and explain their thinking when working with fractions in multiple contexts. They read $\frac{3}{5}$ as “three fifths” and after many experiences with sharing problems, learn that $\frac{3}{5}$ can also be interpreted as “3 divided by 5.”
Example: Sharing 5 objects equally among three shares showing the $\mathbf{5 \div 3 = \frac{5\times1}{3} = \frac{5}{3}}$



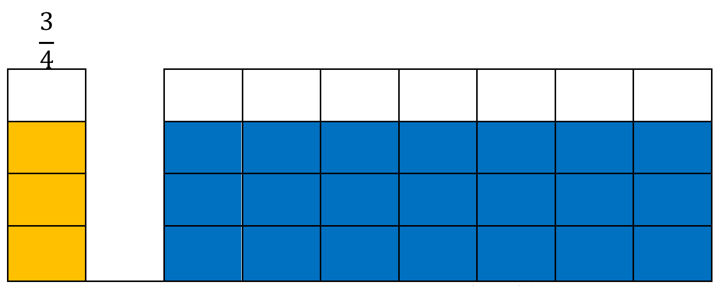
If you divide 5 objects equally among 3 shares, each of the 5 objects should contribute $\frac{1}{3}$ of itself to each share. Thus each share consists of 5 pieces, each of which is 1/3 of an object, so each share is $5 \times \frac{1}{3} = \frac{5}{3}$ of an object.
Please see Progressions for the Common Core State Standards in Mathematics: Number & Operations – Fractions, 3-5.

Instructional Strategies: (5.NF.3-.7)

Connect the meaning of multiplication and division of fractions with whole-number multiplication and division. Consider area models of multiplication and both sharing and measuring models for division.

Ask questions such as, “What does 2×3 mean?” and “What does $12 \div 3$ mean?” Then, follow with questions for multiplication with fractions, such as, “What does $\frac{3}{4} \times \frac{1}{3}$ mean?” “What does $\frac{3}{4} \times 7$ mean?” (7 sets of $\frac{3}{4}$) and What does $7 \times \frac{3}{4}$ mean?” ($\frac{3}{4}$ of a set of 7)

The meaning of $4 \div \frac{1}{2}$ (how many $\frac{1}{2}$ are in 4) and $\frac{1}{2} \div 4$ (how many groups of 4 are in $\frac{1}{2}$) also should be illustrated with models or drawings like:



7 groups of $\frac{3}{4}$ or $\frac{21}{4}$

Encourage students to use models or drawings to multiply or divide with fractions. Begin with students modeling multiplication and division with whole numbers. Have them explain how they used the model or drawing to arrive at the solution.

Models to consider when multiplying or dividing fractions include, but are not limited to: area models using rectangles or squares, fraction strips/bars and sets of counters.

Use calculators or models to explain what happens to the result of multiplying a whole number by a fraction $(3 \times 12, 4 \times 12, 5 \times 12 \ and \ 4 \times 12, 4 \times 13, 4 \times 14, \ldots)$ and when multiplying a fraction by a number greater than 1.

Use calculators or models to explain what happens to the result when dividing a unit fraction by a non-zero whole number $\left(\frac{1}{8} \div 4, \frac{1}{8} \div 8, \ \frac{1}{8} \div \frac{1}{6}, \ldots\right)$ and what happens to the result when dividing a whole number by a unit fraction $\left(4 \div \frac{1}{4}, \ 8 \div \frac{1}{4}, \ 12 \div \frac{1}{4}, \ldots\right)$.

Present problem situations and have students use models and equations to solve the problem. It is important for students to develop understanding of multiplication and division of fractions through contextual situations.

Instructional Resources/Tools

[5.NF Painting a Wall]

[5.NF,MD Converting Fractions of a Unit into a Smaller Unit]

[5.NF How Much Pie?]

[5.NF What is 23 ÷ 5?]

Common Misconceptions: (5.NF.3-7)

Students may believe that multiplication always results in a larger number. Using models when multiplying with fractions will enable students to see that the results will be smaller.

Additionally, students may believe that division always results in a smaller number. Using models when dividing with fractions will enable students to see that the results will be larger.

Domain: Number and Operations –Fractions (NF)

***Cluster:** Apply and extend the previous understandings of multiplication and division to multiply and divide fractions.*

Standard: Grade 5.NF.4

Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product $\left(\frac{a}{b}\right) \times q = \frac{3}{7}$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $\frac{2}{3} \times 4 = \frac{8}{3}$, and create a story context for this equation. Do the same with $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$. (In general, $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1. Make sense of problems and persevere in solving them.
- ✓ MP.2. Reason abstractly and quantitatively.
- ✓ MP.3. Construct viable arguments and critique the reasoning of others.
- ✓ MP.4. Model with mathematics.
- ✓ MP.5. Use appropriate tools strategically.
- ✓ MP.6. Attend to precision.
- ✓ MP.7. Look for and make use of structure.
- ✓ MP.8. Look for and express regularity in repeated reasoning.

Connections: See Grade 5.NF.3

Explanation and Examples:

This standard extends student's work of multiplication from earlier grades. In fourth grade, students worked with recognizing that a fraction such as $\frac{3}{5}$ actually could be represented as 3 pieces that are each one-fifth $\left(3 \times \frac{1}{5}\right)$.

In fifth grade, students are expected to multiply fractions including proper fractions, improper fractions, and mixed numbers. They multiply fractions efficiently and accurately as well as solve problems in both contextual and non-contextual situations.

This standard references both the multiplication of a fraction by a whole number and the multiplication of two fractions. Visual fraction models (area models, tape diagrams, number lines) should be used and created by students during their work with this standard.

As they multiply fractions such as $\frac{3}{5} \times 6$, they can think of the operation in more than one way.

$$3 \times (6 \div 5) \; or \; \left(3 \times \frac{6}{5}\right)$$

$$(3 \times 6) \div 5 \; or \; 18 \div 5 \left(\frac{18}{5}\right)$$

Students create a story problem for $\frac{3}{5} \times 6$ such as,

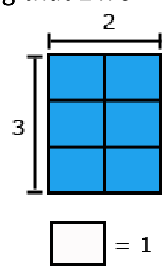
- Isabel had 6 feet of wrapping paper. She used $\frac{3}{5}$ of the paper to wrap some presents. How much does she have left?
- Every day Tim ran $\frac{3}{5}$ of mile. How far did he run after 6 days? (Interpreting this as $6 \times \frac{3}{5}$

Example:
Three-fourths of the class is boys. Two-thirds of the boys are wearing tennis shoes. What fraction of the class are boys with tennis shoes?

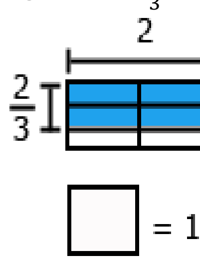
This question is asking what $\frac{2}{3}$ of $\frac{3}{4}$ is, or what is $\frac{2}{3} \times \frac{3}{4}$. In this case you have $\frac{2}{3}$ groups of size $\frac{3}{4}$. (a way to think about it in terms of the language for whole numbers is 4 x 5 you have 4 groups of size 5.
The array model is very transferable from whole number work and then to binomials.

Examples: Building on previous understandings of multiplication

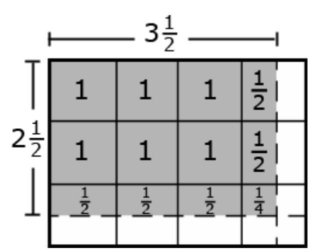
- Rectangle with dimensions of 2 and 3 showing that 2 x 3 = 6.



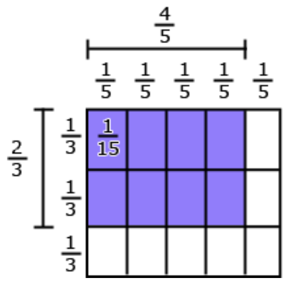
- Rectangle with dimensions of 2 and $\frac{2}{3}$ showing that $2 \times \frac{2}{3} = \frac{4}{3}$



- $2\frac{1}{2}$ groups of $3\frac{1}{2}$:

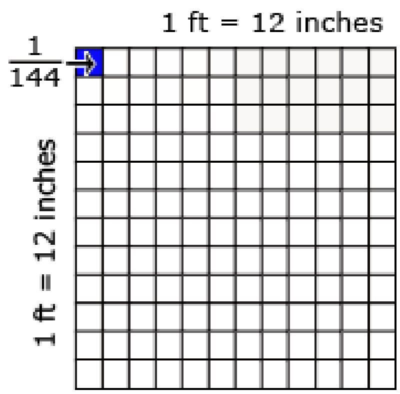


- In solving the problem $\frac{2}{3}$ x $\frac{4}{5}$, students use an area model to visualize it as a 2 by 4 array of small rectangles each of which has side lengths $\frac{1}{3}$ and $\frac{1}{5}$. They reason that $\frac{1}{3} \times \frac{1}{5} = \frac{1}{3 \times 5}$ by counting squares in the entire rectangle, so the area of the shaded area is $(2 \times 4) \times \frac{1}{3 \times 5} = \frac{2 \times 4}{3 \times 5}$. They can explain that the product is less than $\frac{4}{5}$ because they are finding $\frac{2}{3}$ of $\frac{4}{5}$. They can further estimate that the answer must be between $\frac{2}{5}$ and $\frac{4}{5}$ because $\frac{2}{3}$ of $\frac{4}{5}$ is more than $\frac{1}{2}$ of $\frac{4}{5}$ and less than one group of $\frac{4}{5}$.



The area model and the line segments show that the area is the same quantity as the product of the side lengths.

- Larry knows that $\frac{1}{12}$ x $\frac{1}{12}$ is $\frac{1}{144}$. To prove this he makes the following array.
-



Students need to represent problems using various fraction models: Area (rectangle, circle, etc., linear (number line)), and set model and explain their thinking.

Instructional Strategies: **See Grade 5. NF. 3**

Tools/Resources

[5.NF Connor and Makayla Discuss Multiplication]

[5.NF Folding Strips of Paper]

See [engageNY Modules 4 and 5]

Common Misconceptions: See Grade 5.NF. 3

Domain: Number and Operations – Fractions (NF)

Cluster: *Apply and extend previous understandings of multiplication and division to multiply and divide fractions.*

Standard: Grade 5.NF.5

Interpret multiplication as scaling (resizing), by:

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $\frac{a}{b} = \frac{n \times a}{n \times b}$ to the effect of multiplying $\frac{a}{b}$ by 1.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2. Reason abstractly and quantitatively.
- ✓ MP.4. Model with mathematics.
- ✓ MP.6. Attend to precision.
- ✓ MP.7. Look for and make use of structure.

Connections: See Grade 5.NF.3

Explanation and Examples: 5.NF.5a

This standard calls for students to examine the magnitude of products in terms of the relationship between two types of problems. This extends the work with 5.OA.1.

Example 1:
Mrs. Jones teaches in a room that is 60 feet wide and 40 feet long. Mr. Thomas teaches in a room that is half as wide, but has the same length. How do the dimensions and area of Mr. Thomas' classroom compare to Mrs. Jones' room?

Draw a picture to prove your answer.

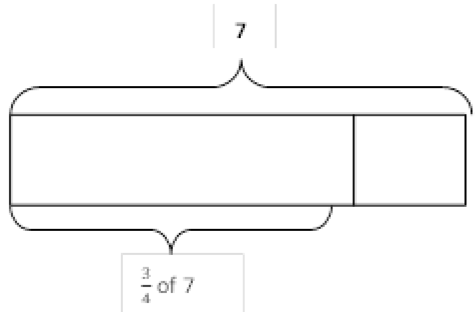
Example 2:
How does the product of 225×60 compare to the product of 225×30?

How do you know?

Since 30 is half of 60, the product of 225×60 will be double or twice as large as the product of 225×30.

Examples:

- $\frac{3}{4} \times 7$ is less than 7 because 7 is multiplied by a factor less than 1 so the product must be less than 7.



5.NF.5b

This standard asks students to examine how numbers change when we multiply by fractions. Students should have ample opportunities to examine both cases in the standard: a) when multiplying by a fraction greater than 1, the number increases and b) when multiplying by a fraction less the one, the number decreases. This standard should be explored and discussed while students are working with 5.NF.4, and **should not** be taught in isolation.

Example:

Mrs. Bennett is planting two flower beds. The first flower bed is 5 meters long and $\frac{6}{5}$ meters wide. The second flower bed is 5 meters long and $\frac{5}{6}$ meters wide. How do the areas of these two flower beds compare? Is the value of the area larger or smaller than 5 square meters? Draw pictures to prove your answer.

Example:

- $2\frac{2}{3}$ x 8 must be more than 8 because 2 groups of 8 is 16 and $2\frac{2}{3}$ is almost 3 groups of 8. So the answer must be close to, but less than 24.
- $\frac{3}{4} = \frac{5\times3}{5\times4}$ because multiplying $\frac{3}{4}$ by $\frac{5}{5}$ is the same as multiplying by 1.

Instructional Strategies: See Grade 5.NF.3

Resources/Tools

5.NF Running a Mile
5.NF Fundraising
5.NF Calculator Trouble
5.NF Comparing a Number and a Product
5.NF Grass Seedlings
5.NF Reasoning about Multiplication

Common Misconceptions: See Grade 5.NF.3

Domain: Number and Operations – Fractions (NF)

Cluster: *Apply and extend previous understandings of multiplication and division to multiply and divide fractions.*

Standard: Grade 5.NF.6

Solve real world problems involving multiplication of fractions and mixed numbers, e.g. by using visual fraction models or equations to represent the problem.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 5.NF.3

Explanation and Examples:

This standard builds on all of the work done in this cluster. Students should be given ample opportunities to use various strategies to solve word problems involving the multiplication of a fraction by a mixed number. This standard could include fraction by a fraction, fraction by a mixed number or mixed number by a mixed number.

Example:

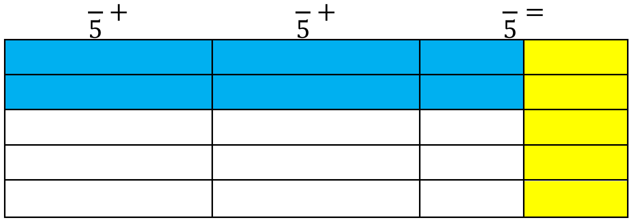
There are $2\frac{1}{2}$ bus loads of students standing in the parking lot. The students are getting ready to go on a field trip. $\frac{2}{5}$ of the students on each bus are girls. How many busses would it take to carry ***only*** the girls?

Student 1

I drew 3 grids and 1 grid represents 1 bus. I cut the third grid in half and I marked out the right half of the third grid, leaving $2\frac{1}{2}$ grids. I then cut each grid into fifths, and shaded two-fifths of each grid to represent the number of girls.

When I added up the shaded pieces, $\frac{2}{5}$ of the 1st and 2nd bus were both shaded, and $\frac{1}{5}$ of the last bus was shaded.

$$\frac{2}{5}+ \quad \frac{2}{5}+ \quad \frac{1}{5}= \quad \frac{5}{5}=1\ whole\ bus$$



Student 2

$2\frac{1}{2} \times \frac{2}{5} =$

I split the $2\frac{1}{2}$ into 2 and $\frac{1}{2}$

$$2 \times \frac{2}{5} = \frac{4}{5}$$

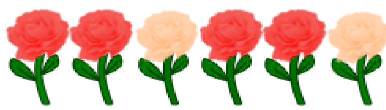
$$\frac{1}{2} \times \frac{2}{5} = \frac{2}{10}$$

I then added $\frac{4}{5}$ and $\frac{2}{10}$. That equals 1 whole bus load.

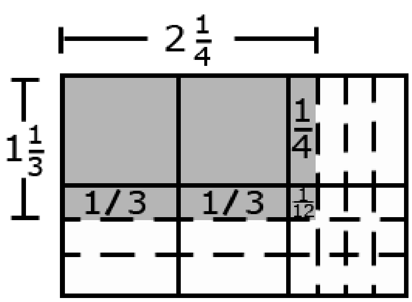
Example:

Evan bought 6 roses for his mother, $\frac{2}{3}$ of them were red. How many red roses were there?

- Using a visual, a student divides the 6 roses into 3 groups and counts how many are in 2 of the 3 groups.



- A student can use an equation to solve
  $\frac{2}{3} \times 6 = \frac{12}{3} = 4\ red\ roses.$
  - Mary and Joe determined that the dimensions of their school flag needed to be $1\frac{1}{3}ft$ by $2\frac{1}{4}ft$. What will be the area of the school flag?
- A student can draw an array to find this product and can also use his or her understanding of decomposing numbers to explain the multiplication. Thinking ahead a student may decide to multiply by $1\frac{1}{3}ft$ instead of $2\frac{1}{4}ft$.



The explanation may include the following:

First, I am going to multiply $2\frac{1}{4}$ by 1 and then by $\frac{1}{3}$.

When I multiply $2\frac{1}{4}$ by 1, it equals $2\frac{1}{4}$.

Now I have to multiply $2\frac{1}{4}$ by $\frac{1}{3}$.

$\frac{1}{3}$ times 2 is $\frac{2}{3}$.

$\frac{1}{3}$ times $\frac{1}{4}$ is $\frac{1}{12}$.

So the answer is $2\frac{1}{4} + \frac{2}{3} + \frac{1}{12}$ or $2\frac{3}{12} + \frac{8}{12} + \frac{1}{12} = 2\frac{12}{12} = 3$

Instructional Strategies: See Grade 5.NF.3

Resource/Tools

[5.NF Running to School]
[5.NF Drinking Juice]
[5.NF Half of a Recipe]
[5.NF Making Cookies]
[5.NF To Multiply or not to multiply?]

Common Misconceptions: See Grade 5.NF.3

Domain: Number and Operations – Fractions (NF)

Cluster: *Apply and extend previous understandings of multiplication and division to multiply and divide fractions.*

Standard: Grade 5. NF.7

Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.)

1. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for* $\frac{1}{3} \div 4$*, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $\frac{1}{3} \div 4 = \frac{1}{12}$ *because* $\frac{1}{12} \times 4 = \frac{1}{3}$.
2. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for* $4 \div \frac{1}{5}$*, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that* $4 \div \frac{1}{5} = 20$ *because* $20 \times \frac{1}{5} = 4$.

Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share* $\frac{1}{2}\, lb$ *of chocolate equally? How many* $\frac{1}{3}\, cup$ *servings are in 2 cups of raisins?*

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: See Grade 5.NF.3

Explanation and Examples:

5.NF.7 is the first time that students are dividing with fractions. In fourth grade students divided whole numbers, and multiplied a whole number by a fraction. In fifth grade, students experience division problems with whole number divisors and unit fraction dividends (fractions with a numerator of 1) or with unit fraction divisors and whole number dividends.

For example, the fraction $\frac{3}{5}$ is 3 copies of the *unit fraction* $\frac{1}{5}$. $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5} = \frac{1}{5} \times 3 \; or \; 3 \times \frac{1}{5}$.

Students extend their understanding of the meaning of fractions, how many unit fractions are in a whole, and their understanding of multiplication and division as involving equal groups or shares and the number of objects in each group/share. In sixth grade, they will use this foundational understanding to divide into and by more complex fractions and develop abstract methods of dividing by fractions.

5.NF.7a This standard asks students to work with story contexts where a *unit fraction* is divided by a non-zero whole number. Students should use various fraction models and reasoning about fractions.

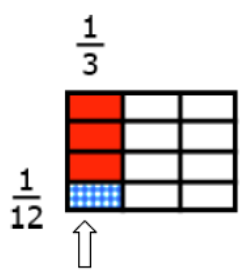
Example:
You have $\frac{1}{8}$ of a bag of pens and you need to share them among 3 people. How much of the bag does each person get?

5.NF.7b This standard calls for students to create story contexts and visual fraction models for division situations where a whole number is being divided by a unit fraction.

Example:
Create a story context for $5 \div \frac{1}{6}$. Find your answer and then draw a picture to prove your answer and use multiplication to reason about whether your answer makes sense. How many 1/6 are there in 5?

Example:
Knowing the number of groups/shares and finding how many/much in each group/share.
Four students sitting at a table were given $\frac{1}{3}$ of a pan of brownies to share. How much of a pan will each student get if they share the pan of brownies equally?

The diagram shows the $\frac{1}{3}$ pan divided into 4 equal shares with each share equaling $\frac{1}{12}$ of the pan.



5.NF.7c Extends students' work from other standards in 5.NF.7. Students should continue to use visual fraction models and reasoning to solve these real-world problems.

Example:
How many $\frac{1}{3}$-cup servings are in 2 cups of raisins?

5.NF.7c Extends students' work from other standards in 5.NF.7. Students should continue to use visual fraction models and reasoning to solve these real-world problems.

Example:

How many $\frac{1}{3}$-cup servings are in 2 cups of raisins?

Student

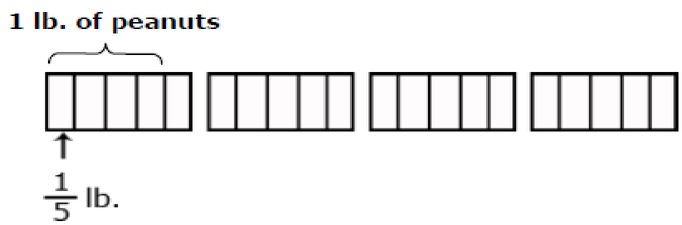
I know that there are three $\frac{1}{3}$ cup servings in 1 cup of raisins. Therefore, there are 6 servings in 2 cups of raisins. I can also show this since 2 divided by $\frac{1}{3} = 2 \times 3 = 6$ servings of raisins.

Example:

Knowing how many in each group/share and finding how many groups/shares

- Angelo has 4 lbs of peanuts. He wants to give each of his friends $\frac{1}{5}\,lb$. How many friends can receive $\frac{1}{5}\,lb$ of peanuts?

A diagram for $4 \div \frac{1}{5}$ is shown below. Students explain that since there are five fifths in one whole, there must be 20 fifths in 4 lbs.



Example:

- How much rice will each person get if 3 people share $\frac{1}{2}\,lb$ of rice equally?

$$\frac{1}{2} \div 3 = \frac{3}{6} \div 3 = \frac{1}{6}$$

  - A student may think or draw ½ and cut it into 3 equal groups then determine that each of those part is $\frac{1}{6}$.
  - A student may think of $\frac{1}{2}$ as equivalent to $\frac{3}{6}$. $\frac{3}{6}$ divided by 3 is $\frac{1}{6}$.

It's important that students *represent* the problems they are solving, have a visual image of the "why" behind the algorithm and can explain their reasoning.

Instructional Strategies: See Grade 5.NF.3

Resources/Tools

5.NF Dividing by One-Half
5.NF How many servings of oatmeal?
5.NF Banana Pudding
5.NF Painting a room
5.NF How many marbles?
5.NF Origami Stars
5.NF How many marbles?
5.NF Salad Dressing
5.NF Standing in Line

Common Misconceptions: See Grade 5.NF.3

Domain: Measurement and Data (MD)

***Cluster:** Converts like measurement units within a given measurement system.*

Standard: Grade 5.MD.1

Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.

Connections:

This cluster is connected to:

- Grade 5 Critical Area of Focus #2, **Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations.**
- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit (Grade 4 MD 1

Explanation and Examples:

This standard calls for students to convert measurements within the same system of measurement in the context of multi-step, real-world problems.

Both customary and standard measurement systems are included; students worked with both metric and customary units of length in second grade. In third grade, students work with metric units of mass and liquid volume.

In fourth grade, students work with both systems and begin conversions within systems in length, mass and volume. Fifth graders build on their prior knowledge of related measurement units to determine equivalent measurements. Prior to making actual conversions, they examine the units to be converted, determine if the converted amount will be more or less than the original unit, and explain their reasoning. They use several strategies to convert measurements. When converting metric measurement, students apply their understanding of place value and decimals.

Students' work with conversions within the metric system (5.MD.1) provides opportunities for practical applications of place value understanding and supports major work at the grade in the cluster "Understand the place value system". (5.NBT.1)

Example:

100 cm = 1 meter.

Instructional Strategies:

Students should gain ease in converting units of measures in equivalent forms within the same system. To convert from one unit to another unit, the **relationship** between the units must be known. In order for students to have a better understanding of the relationships between units, they need to use measuring tools in class. The number of units must relate to the size of the unit.

For example, students have discovered that there are 12 inches in 1 foot and 3 feet in 1 yard. This understanding is needed to convert inches to yards. Using 12-inch rulers and yardsticks, students can see that three of the 12-inch rulers are equivalent to one yardstick (3×12 inches = 36 inches; 36 inches = 1 yard). Using this knowledge, students can decide whether to multiply or divide when making conversions.

Once students have an understanding of the relationships between units and how to do conversions, they are ready to solve multi-step problems that require conversions within the same system. Allow students to discuss methods used in solving the problems. Begin with problems that allow for renaming the units to represent the solution before using problems that require renaming to find the solution.

Instructional Resources/Tools

Yardsticks(meter sticks) and rulers (marked with customary and metric units)
Teaspoons and tablespoons
Graduated measuring cups (marked with customary and metric units)

For detailed information see: Learning Progressions on Measurement and Data

5.NF,MD Converting Fractions of a Unit into a Smaller Unit
5.MD Minutes and Days

"Discovering Gallon Man" NCTM.org, Illuminations. Students experiment with units of liquid measure used in the customary system of measurement. They practice making volume conversions in the customary system.

"Do You Measure Up?" NCTM.org, Illuminations. Students learn the basics of the metric system. They identify which units of measurement are used to measure specific objects, and they learn to convert between units within the same system.

Common Misconceptions:

When solving problems that require renaming units, students use their knowledge of renaming the numbers as with whole numbers. Students need to pay attention to the unit of measurement which dictates the renaming and the number to use. The same procedures used in renaming whole numbers should not be taught when solving problems involving measurement conversions. For example, when subtracting 5 inches from 2 feet, students may take one foot from the 2 feet and use it as 10 inches. Since there were no inches with the 2 feet, they put 1 with 0 inches and make it 10 inches.

2 feet	*is thought of as*	2 feet 0 inches	*becomes*	1 foot 10 inches
- 5 inches		- 5 inches		- 5 inches
				1 foot 5 inches

Domain: Measurement and Data (MD)

Cluster:* *Represent and interpret data.

Standard: Grade 5.MD.2

Make a line plot to display a data set of measurements in fractions of a unit$\left(\frac{1}{2},\frac{1}{4},\frac{1}{8}\right)$. Use operations on fractions for this grade to solve problems involving information presented in line plots.
For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections:

This cluster is connected to:

- Grade 5 Critical Area of Focus #1, **Developing fluency with addition and subtraction of fractions and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions)**.
- Use equivalent fractions as a strategy to add and subtract fractions (Grade 5 NF 1 and 2).
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions (Grade 5 NF 4 and 7).

Explanation and Examples:

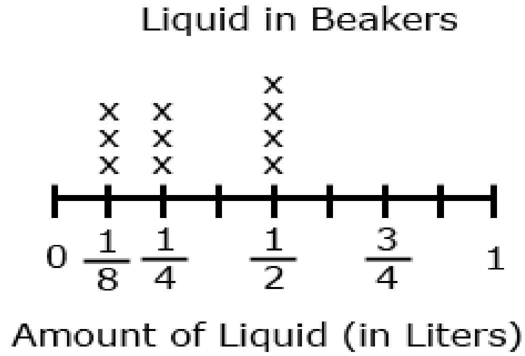
This standard provides a context for students to work with fractions by measuring objects to one-eighth of a unit. This includes length, mass, and liquid volume. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.

Example:

Students measured objects in their desk to the nearest $\frac{1}{2},\frac{1}{4}, or\ \frac{1}{8}$ of an inch then displayed data collected on a line plot. How many object measured $\frac{1}{4}$? $\frac{1}{2}$? If you put all the objects together end to end what would be the total length of all the objects?

Example:

Ten beakers, measured in quarts, are filled with a liquid.



The line plot above shows the amount of liquid in liters in 10 beakers. If the liquid is redistributed equally, how much liquid would each beaker have? (This amount is the mean.)

Students apply their understanding of operations with fractions. They use either addition and/or multiplication to determine the total number of liters in the beakers. Then the sum of the liters is shared evenly among the ten beakers.

Instructional Strategies:

Using a line plot to solve problems involving operations with unit fractions now includes multiplication and division. Revisit using a number line to solve multiplication and division problems with whole numbers. In addition to knowing how to use a number line to solve problems, students also need to know which operation to use to solve problems.

Use the tables for common addition and subtraction, and multiplication and division situations (Table 1 and Table 2 in the appendix as a guide to the types of problems students need to solve without specifying the type of problem. Allow students to share methods used to solve the problems. Also have students create problems to show their understanding of the meaning of each operation.

Resources/Tools:

"Fractions in Every Day Life", NCTM.org, Illuminations". This activity enables students to apply their knowledge about fractions to a real-life situation. It also provides a good way for teachers to assess students' working knowledge of fraction multiplication and division. Students should have prior knowledge of adding, subtracting, multiplying, and dividing fractions before participating in this activity. This will help students to think about how they use fractions in their lives, sometimes without even realizing it. The basic idea behind this activity is to use a recipe and alter it to serve larger or smaller portions.

http://mathforum.org/paths/fractions/frac.recipe.html

5.NF Fractions on a Line Plot

Common Misconceptions:

Some students will need additional help in understanding that it is possible to graph with fractions. This may be their first experience.

Students may not always establish the correct locations on the x-coordinate.

Domain: Measurement and Data (MD)

***Cluster:* Geometric measurement: understand concepts of volume and relate volume to multiplication and addition.**

Standard: Grade 5.MD.3-5

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
   a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
   b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.
4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.
   a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
   b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems.
   c. Recognize volume as additive. Find volumes of solid figures composed of two non- overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of other.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.
- ✓ MP.8 Look for and express regularity in repeated reasoning.

Connections: 5.MD.3-5

This cluster is connected to:

- Grade 5 Critical Area of Focus #3, **Developing understanding of volume.**
- Use place value understanding and properties of operations to perform multi-digit arithmetic (Grade 4 NBT 5).

Explanation and Examples: (5.MD.3-5)

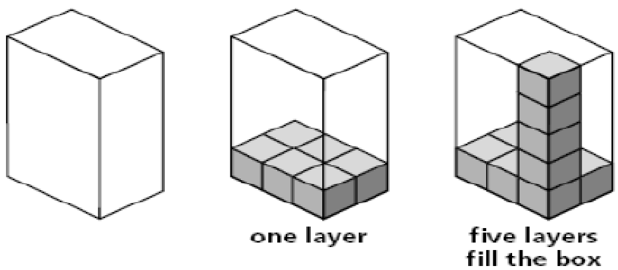
These standards represent the first time that students begin exploring the concept of volume. Their prior experiences with volume were restricted to liquid volume. In third grade, students begin working with area and covering spaces. The concept of volume should be extended from area with the idea that students are covering an area (the bottom of cube) with a layer of unit cubes and then adding layers of unit cubes on top of bottom layer *(see picture below).*

Students should have ample experiences with concrete manipulatives before moving to pictorial representations. As students develop their understanding of volume they recognize that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. This cube has a length of 1 unit, a width of 1 unit and a height of 1 unit and is called a cubic unit.

This cubic unit is written with an exponent of 3 (e.g., in^3, m^3). Students connect this notation to their understanding of powers of 10 in our place value system.

In grade three students measured and estimated liquid volume and worked with area measurement. At grade five, the concept of volume can be extended from area by relating earlier work covering an area to the bottom of cube with a layer of unit cubes and then adding layers of unit cubes on top of bottom layer. Models of cubic inches, centimeters, cubic feet, etc. are helpful in developing an image of a cubic unit. For example: Student's estimate how many cubic yards would be needed to fill the classroom or how many cubic centimeters would be needed to fill a pencil box.

Example:



(3 x 2) represented by first layer
(3 x 2) x 5 represented by number of 3 x 2 layers
(3 x 2) + (3 x 2) + (3 x 2) + (3 x 2)+ (3 x 2) =
6 + 6 + 6 + 6 + 6 + 6 = 30
6 representing the size/area of one layer

5.MD.4

Students understand that same sized cubic units are used to measure volume. They select appropriate units to measure volume. For example, they make a distinction between which units are more appropriate for measuring the volume of a gym and the volume of a box of books.

They can also improvise a cubic unit using any unit as a length (e.g., the length of their pencil). Students can apply these ideas by filling containers with cubic units (wooden cubes) to find the volume. They may also use drawings or interactive computer software to simulate the same filling process.

5.MD.5 a-b

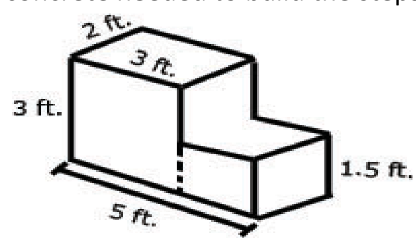
These standards involve finding the volume of right rectangular prisms *(as shown in picture on previous page)*. Students should have experiences to describe and reason about **why** the formula is true. Specifically, that they are covering the bottom of a right rectangular prism (length x width) with multiple layers (height). Therefore, the formula (length x width x height) is an extension of the formula for the area of a rectangle.

Examples:

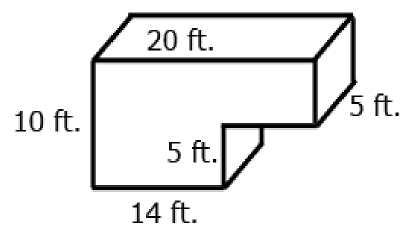
- When given 24 cubes, students make as many rectangular prisms as possible with a volume of 24 cubic units. Students build the prisms and record possible dimensions.

Length	Width	Height
1	2	12
2	2	6
4	2	3
8	3	1

- Students determine the volume of concrete needed to build the steps in the diagram below.



- A homeowner is building a swimming pool and needs to calculate the volume of water needed to fill the pool. The design of the pool is shown in the illustration below.



5.MD.5c

This standard calls for students to extend their work with the area of composite figures into the context of volume. Students should be given concrete experiences of breaking apart (decomposing) 3-dimensional figures into right rectangular prisms in order to find the volume of the entire 3-dimensional figure.

Students need multiple opportunities to measure volume by filling rectangular prisms with cubes and looking at the **relationship** between the total volume and the area of the base.

They derive the volume formula (volume equals the area of the base times the height) and explore how this idea would apply to other prisms.

Students use the associative property of multiplication and decomposition of numbers using factors to investigate rectangular prisms with a given number of cubic units.

Instructional Strategies:

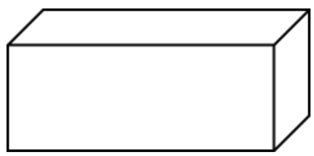
5.MD.3-5:

Volume refers to the amount of space that an object takes up and is measured in cubic units such as cubic inches or cubic centimeters.

Provide students with opportunities to find the volume of rectangular prisms by counting unit cubes, in metric and standard units of measure, before the formula is presented. Multiple opportunities are needed for students to develop the formula for the volume of a rectangular prism with activities similar to the one described below.

Give students one block (a 1- or 2- cubic centimeter or cubic-inch cube), a ruler with the appropriate measure based on the type of cube, and a small rectangular box. Ask students to determine the number of cubes needed to fill the box. Have students share their strategies with the class using words, drawings or numbers. Allow them to confirm the volume of the box by filling the box with cubes of the same size.

By stacking geometric solids with cubic units in layers, students can begin understanding the concept of how *addition plays a part in finding volume*. This will lead to an understanding of the formula for the volume of a right rectangular prism, b x h, where b is the area of the base. A right rectangular prism has three pairs of parallel faces that are all rectangles.



Have students build a prism in layers. Then, have students determine the number of cubes in the bottom layer and share their strategies. Students should use multiplication based on their knowledge of arrays and its use in multiplying two whole numbers.

Ask what strategies can be used to determine the volume of the prism based on the number of cubes in the bottom layer. Expect responses such as “adding the same number of cubes in each layer as were on the bottom layer” or multiply the number of cubes in one layer times the number of layers.

Instructional Resources/Tools

See engageNY Module 5

5.MD,OA You Can Multiply Three Numbers in Any Order
5.MD Using Volume to Understand the Associative Property of Multiplication
5.MD Cari's Aquarium

See “Varying Volumes”, NCSM, Great Tasks for Mathematics K-5, (2013)
Learning Progressions on Measurement and Data

Common Misconceptions:

Students are unsure as to which units to use to measure volume because they are not sure what they are measuring. Also, they may confuse the need to find volume with area.

Domain: Geometry (G)

Cluster: *Graph points on the coordinate plane to solve real-world and mathematical problems.*

Standard: Grade 5.G.1

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: 5.G.1-2.

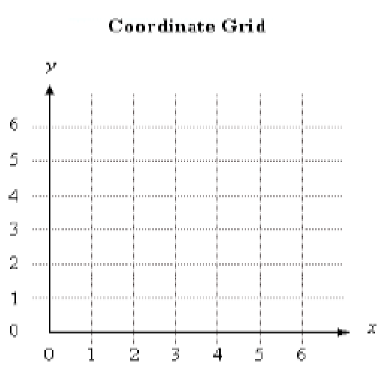
This cluster goes beyond the Grade 5 Critical Areas of Focus to address Modeling numerical relationships with the coordinate plane.

Explanation and Examples:

5.G1 and 5.G.2

These standards deal with only the first quadrant (positive numbers in the coordinate plane.

5.G.1 Examples:



Example:
Connect these points in order on the coordinate grid below:
(2, 2) (2, 4) (2, 6) (2, 8) (4, 5) (6, 8) (6, 6) (6, 4) and (6, 2).

What letter is formed on the grid?

Solution: "M" is formed.

Example:

Plot these points on a coordinate grid.

- Point A: (2,6)
- Point B: (4,6)
- Point C: (6,3)
- Point D: (2,3)

Connect the points in order. Make sure to connect Point D back to Point A.

1. What geometric figure is formed? What attributes did you use to identify it?
2. What line segments in this figure are parallel?
3. What line segments in this figure are perpendicular?

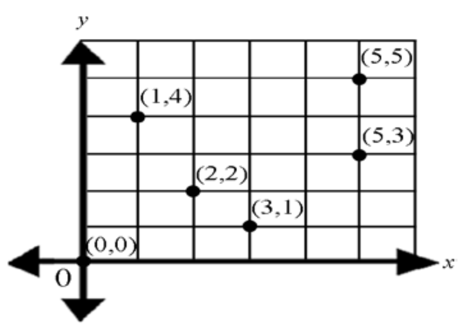
Solutions: trapezoid, line segments AB and DC are parallel, segments AD and DC are perpendicular

Example:

Emanuel draws a line segment from (1, 3) to (8, 10). He then draws a line segment from (0, 2) to (7, 9). If he wants to draw another line segment that is parallel to those two segments what points will he use?

Examples:

- Students can use a classroom size coordinate system to physically locate the coordinate point (5, 3) by starting at the origin point (0,0), walking 5 units along the x axis to find the first number in the pair (5), and then walking up 3 units for the second number in the pair (3). The ordered pair names a point in the plane.



- Graph and label the points below in a coordinate system.
  - A (0, 0)
  - B (5, 1)
  - C (0, 6)
  - D (2.5, 6)
  - E (6, 2)
  - F (4, 1)
  - G (3,0)

Instructional Strategies:

Students need to understand the underlying structure of the coordinate system and see how axes make it possible to locate points anywhere on a coordinate plane. This is the first time students are working with coordinate planes, and only in the first quadrant. It is important that students create the coordinate grid themselves. This can be related to two number lines and reliance on previous experiences with moving along a number line.

Multiple experiences with plotting points are needed. Provide points plotted on a grid and have students name and write the ordered pair. Have students **describe** how to get to the location. Encourage students to articulate directions, attending to precision as they plot points.

Present real-world and mathematical problems and have students graph points in the first quadrant of the coordinate plane. Gathering and graphing data is a valuable experience for students. It helps them to develop an understanding of coordinates and what the overall graph represents. Students also need to analyze the graph by interpreting the coordinate values in the context of the situation.

Instructional Resources/Tools

For detailed information see, Learning Progressions for Geometry

http://www.learner.org/series/modules/express/videos/video_clips.html?type=1&subject=math&practice=structure

5.G Battle Ship Using Grid Paper

”*Finding Your Way Around”*, NCTM, Illuminations. Students explore two-dimensional space via an activity in which they navigate the coordinate plane.

“Describe the Graph”’NCTM.org, Illuminations. In this lesson, students will review plotting points and labeling axes. Students generate a set of random points all located in the first quadrant.

Common Misconceptions:

5.G.1-2

When playing games with coordinates or looking at maps, students may think the order in plotting a coordinate point is not important. Have students plot points so that the position of the coordinates is switched. For example, have students plot (3, 4) and (4, 3) and discuss the order used to plot the points. Have students create directions for others to follow so that they become aware of the importance of direction and distance.

Domain: Geometry (G)

***Cluster**: Graph points on the coordinate plane and solve real-world and mathematical problems.*

Standard: Grade 5.G.2

Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation**.**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.1 Make sense of problems and persevere in solving them.
- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.4 Model with mathematics.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

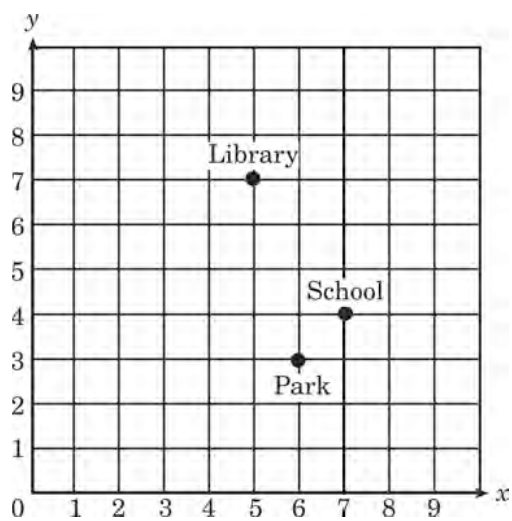
Connections: See Grade 5.G.1

Explanation and Examples:

This standard references real-world and mathematical problems, including the traveling from one point to another and identifying the coordinates of missing points in geometric figures, such as squares, rectangles, and parallelograms.

Example:

Using the coordinate grid, which an ordered pair represents the location of the School? Explain a possible path from the school to the library.



Examples:

Barb has saved $20. She earns $8 for each hour she works.

- o If Barb saves all of her money, how much will she have after working 3 hours? 5 hours? 10 hours?
- o Create a graph that shows the relationship between the hours Barb worked and the amount of money she has saved.
- o What other information do you know from analyzing the graph?

Use the graph below to determine how much money Barb makes after working exactly 9

Instructional Strategies: **See Grade 5.G.1**

Resources/Tools:
5.G Meerkat Coordinate Plane Task

Common Misconceptions: See Grade 5.G.1

Domain: Geometry (G)

***Cluster:** Classify two-dimensional figures into categories based on their properties.*

Standard: Grade 5.G 3

Understand that attributes belonging to a category of two dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles**.**

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: 5.G.3-4

This cluster is connected to:

- Grade 5 Critical Area of Focus #3, Developing understanding of volume.
  Reason with shapes and their attributes (Grade 3 G 1.
- Draw and identify lines and angles, and classify shapes by properties of their lines and angles (Grade 4 G 1–2).

Explanation and Examples:

This standard calls for students to reason about the attributes (properties) of shapes. Student should have experiences discussing the property of shapes and explaining their reasoning.

Geometric properties include properties of sides (parallel, perpendicular, congruent), properties of angles (type, measurement, congruent), and properties of symmetry (point and line).

Example:
Examine whether all quadrilaterals have right angles. Give examples and non-examples.

Example:
If he opposite sides on a parallelogram are parallel and congruent, then rectangles are parallelograms.
A sample of questions that might be posed to students include:

- A parallelogram has 4 sides with both sets of opposite sides parallel. What types of quadrilaterals are parallelograms? Explain.
- Regular polygons have all of their sides and angles congruent. Name or draw some regular polygons. Explain your drawings.
- All rectangles have 4 right angles. Squares have 4 right angles so they are also rectangles. True or False? Explain your reasoning.
- A trapezoid has 2 sides parallel so it must be a parallelogram. True or False? Explain your reasoning.

Instructional Strategies: **5.G.3-4**
This cluster builds from Grade 3 when students described, analyzed and compared properties of two-dimensional shapes. They compared and classified shapes by their sides and angles, and connected these with definitions of shapes.

In Grade 4 students built, drew and analyzed two-dimensional shapes to deepen their understanding of the properties of two-dimensional shapes. They looked at the presence or absence of parallel and perpendicular lines or the presence or absence of angles of a specified size to classify two-dimensional shapes.

Now, students classify two-dimensional shapes in a hierarchy based on properties. Details learned in earlier grades need to be used in the descriptions of the attributes of shapes. The more ways that students can classify and discriminate shapes, the better they can understand them. The shapes are not limited to quadrilaterals.

Students can use graphic organizers such as flow charts or T-charts to compare and contrast the attributes of geometric figures. Have students create a T-chart with a shape on each side. Have them list attributes of the shapes, such as number of side, number of angles, types of lines, etc. they need to determine what's alike or different about the two shapes to get a larger classification for the shapes and be able to explain these properties.

Pose questions such as, "Why is a square always a rectangle?" and "Why is a rectangle not always a square?" Expect students to use precision in justifying and explaining their reasoning.

Resources/Tools
See engageNY Module 5

"Geometric Solids", NCTM.org, Illuminations has a is a tool that allows the student to learn about various geometric solids and their properties. The shapes can be manipulated, colored to explore the number of faces, edges, and vertices and they can be to investigate question such as: what is the relationship between the number of faces, vertices and edges?

Common Misconceptions: 5.G.3-4
Students think that when describing geometric shapes and placing them in subcategories, the last category is the only classification that can be used.

Domain: Geometry (G)

***Cluster:** Classify two-dimensional figures into categories based on their properties.*

Standard: Grade 5.G.4

Classify two dimensional figures in a hierarchy based on properties.

Suggested Standards for Mathematical Practice (MP):

- ✓ MP.2 Reason abstractly and quantitatively.
- ✓ MP.3 Construct viable arguments and critique the reasoning of others.
- ✓ MP.5 Use appropriate tools strategically.
- ✓ MP.6 Attend to precision.
- ✓ MP.7 Look for and make use of structure.

Connections: See Grade 5.G.3

Explanation and Examples:

This standard builds on what was done in 4th grade. Figures from previous grades: **polygon, rhombus/rhombi, rectangle, square, triangle, quadrilateral, pentagon, hexagon, cube, trapezoid, half/quarter circle, circle**

Properties of figure may include:

- Properties of sides—parallel, perpendicular, congruent, number of sides
- Properties of angles—types of angles, congruent

Examples:

- A right triangle can be both scalene and isosceles, but not equilateral.
- A scalene triangle can be right, acute and obtuse.

Triangles can be classified by:

- Angles
  - Right: The triangle has one angle that measures 90º.
  - Acute: The triangle has exactly three angles that measure between 0º and 90º.
  - Obtuse: The triangle has exactly one angle that measures greater than 90º and less than 180º.
- Sides
  - Equilateral: All sides of the triangle are the same length.
  - Isosceles: At least two sides of the triangle are the same length.
  - Scalene: No sides of the triangle are the same length.

Example:

Create a Hierarchy Diagram using the following terms:

polygons – a closed plane figure formed from line segments that meet only at their endpoints.

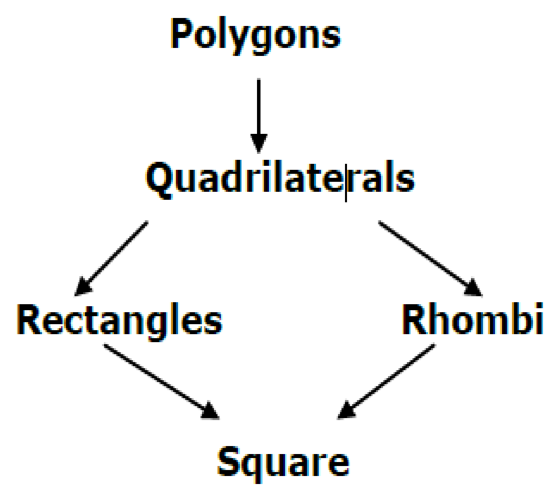
quadrilaterals - a four-sided polygon.

rectangles - a quadrilateral with two pairs of congruent parallel sides and four right angles.

rhombi – a parallelogram with all four sides equal in length.

square – a parallelogram with four congruent sides and four right angles.

Possible student solutions:



quadrilaterals - a four-sided polygon.

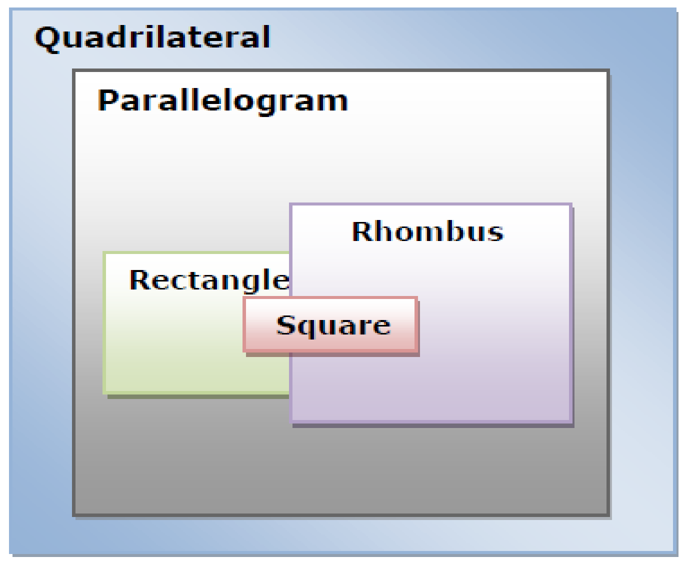
parallelogram: a quadrilateral with two pairs of parallel and congruent sides.

rectangles - a quadrilateral with two pairs of congruent parallel sides and four right angles.

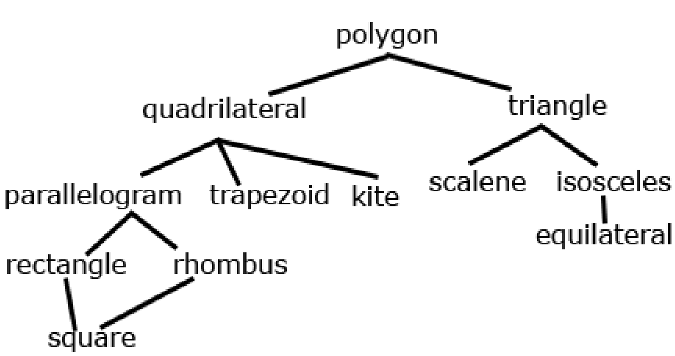
rhombus – a parallelogram with all four sides equal in length.

square – a parallelogram with four congruent sides and four right angles.

Possible student solution:



Example:



Students should be able to reason about the attributes of shapes by examining:

- What are ways to classify triangles?
- Why can't trapezoids and kites be classified as parallelograms?
- Which quadrilaterals have opposite angles congruent and why is this true of certain quadrilaterals?, and
- How many lines of symmetry does a regular polygon have?

Instructional Strategies: **See Grade 5.G.3**

Resources/Tools

5.G What is a Trapezoid? (Part 2)

Common Misconceptions: See Grade 5.G.3

Appendix

TABLE 1. Common Addition and Subtraction Situations[6]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	**Addend Unknown**	**Both Addends Unknown[1]**
Put Together / Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$ or $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$ or $5 = 5 + 0$ $5 = 1 + 4$ or $5 = 4 + 1$ $5 = 2 + 3$ or $5 = 3 + 2$
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$ or $5 - 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$ or $3 + 2 = ?$	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ or $? + 3 = 5$

[1]These *take apart* situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean *makes* or *results in* but always does mean *is the same number as*.

[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation especially for small numbers less than or equal to 10.

[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using *more* for the bigger unknown and using *less* for the smaller unknown). The other versions are more difficult.

[6]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

TABLE 2. Common Multiplication and Division Situations[7]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ And $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*: You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*: You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*: You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*: What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*: A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*: A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*: A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*: A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? *Measurement example*: A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[7]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

TABLE 3. The Properties of Operations

Associative property of addition	$(a+b)+c=a+(b+c)$
Commutative property of addition	$a+b=b+a$
Additive identity property of 0	$a+0=0+a=a$
Existence of additive inverses	For every (a) there exists$(-a)$ so that $a+(-a)=(-a)+a=0$
Associative property of multiplication	$(a\times b)\times c=a\times(b\times c)$
Commutative property of multiplication	$a\times b=b\times a$
Multiplicative identity property of 1	$a\times 1=1\times a=a$
Existence of multiplicative inverses	For every $a\neq 0$ there exists $\frac{1}{a}$ so that $a\times\frac{1}{a}=\frac{1}{a}\times a=1$
Distributive property of multiplication over addition	$a\times(b+c)=a\times b+a\times c$

Here *a*, *b* and *c* stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

*TABLE 4. The Properties of Equality**

Reflexive property of equality	$a=a$
Symmetric property of equality	If $a=b$ then $b=a$
Transitive property of equality	If $a=b$ and $b=c$, then $a=c$
Addition property of equality	If $a=b$ then $a+c=b+c$
Subtraction property of equality	If $a=b$ then $a-c=b-c$
Multiplication property of equality	If $a=b$ then $a\times c=b\times c$
Division property of equality	If $a=b$ and $c\neq 0$ then $a\div c=b\div c$
Substitution property of equality	If $a=b$ then b may be substituted for a in any expression containing a.

Here *a*, *b* and *c* stand for arbitrary numbers in the rational, real, or complex number systems.

TABLE 5. The Properties of Inequality

Exactly one of the following is true: $a<b, a=b, a>b$.
If $a>b$ and $b>c$ then $a>c$
If $a>b$ then $b<a$
If $a>b$ then $-a<-b$
If $a>b$ then $a\pm c>b\pm c$
If $a>b$ and $c>0$ then $a\times c>b\times c$
If $a>b$ and $c<0$ then $a\times c<b\times c$
If $a>b$ and $c>0$ then $a\div c>b\div c$
If $a>b$ and $c<0$ then $a\div c<b\div c$

Here *a*, *b* and *c* stand for arbitrary numbers in the rational or real number systems.

Table 6. Cognitive Rigor Matrix/Depth of Knowledge (DOK)

The Common Core State Standards require high-level cognitive demand asking students to demonstrate deeper conceptual understanding through the application of content knowledge and skills to new situations and sustained tasks. For each Assessment Target the depth(s) of knowledge (DOK) that the student needs to bring to the item/task will be identified, using the Cognitive Rigor Matrix shown below.

Depth of Thinking (Webb)+ Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	• Recall conversions, terms, facts			
Understand	• Evaluate an expression • Locate points on a grid or number on number line • Solve a one-step problem • Represent math relationships in words, pictures, or symbols	• Specify, explain relationships • Make basic inferences or logical predictions from data/observations • Use models/diagrams to explain concepts • Make and explain estimates	• Use concepts to solve non-routine problems • Use supporting evidence to justify conjectures, generalize, or connect ideas • Explain reasoning when more than one response is possible • Explain phenomena in terms of concepts	• Relate mathematical concepts to other content areas, other domains • Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	• Follow simple procedures • Calculate, measure, apply a rule (e.g., rounding) • Apply algorithm or formula • Solve linear equations • Make conversions	• Select a procedure and perform it • Solve routine problem applying multiple concepts or decision points • Retrieve information to solve a problem • Translate between representations	• Design investigation for a specific purpose or research question • Use reasoning, planning, and supporting evidence • Translate between problem & symbolic notation when not a direct translation	• Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	• Retrieve information from a table or graph to answer a question • Identify a pattern/trend	• Categorize data, figures • Organize, order data • Select appropriate graph and organize & display data • Interpret data from a simple graph • Extend a pattern	• Compare information within or across data sets or texts • Analyze and draw conclusions from data, citing evidence • Generalize a pattern • Interpret data from complex graph	• Analyze multiple sources of evidence or data sets
Evaluate			• Cite evidence and develop a logical argument • Compare/contrast solution methods • Verify reasonableness	• Apply understanding in a novel way, provide argument or justification for the new application
Create	• Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	• Generate conjectures or hypotheses based on observations or prior knowledge and experience	• Develop an alternative solution • Synthesize information within one data set	• Synthesize information across multiple sources or data sets • Design a model to inform and solve a practical or abstract situation

References, Resources, and Links

1. Achieve the Core. http://www.achievethecore.org/.
2. Bolam, R., McMahon, A., Stoll, L., Thomas, S., & Wallace, M. (2005). Creating and sustaining professional learning communities. *Research Report Number 637*. General Teaching Council for England. London, England: Department for Education and Skills.
3. Croft, A., Coggshall, J. G., Dolan, M., Powers, E., with Killion, J. (2010). *Job-embedded professional development: What it is, who is responsible, and how to get it done well.* National Comprehensive Center for Teacher Quality. Retrieved March 11, 2013, from http://www.tqsource.org/
4. Darling-Hammond, L., Wei, R.C., Andree, A., Richardson, N., & Orphanos, S. (2009). *Professional learning in the learning profession: a status report on teacher development in the United States and abroad.* Oxford, OH: National Staff Development Council and the School Redesign Network at Stanford University.
5. Garet, M. S., Porter, A. C., Desimone, L., Birman, B., & Yoon, K. (2001). What makes professional development effective? *American Educational Research Journal,* 38(4), 915–945. Retrieved March 11, 2013, from www.aztla.asu.edu/ProfDev1.pdf
6. Guskey, T. (2000). *Evaluating professional development.* Thousand Oaks, CA: Corwin Press.
7. Illustrative Mathematics. http://www.illustrativemathematics.org/.
8. Inside Mathematics. http://www.insidemathematics.org/.
9. Kanold, T. & Larson, M. (2012). *Common Core Mathematics in a PLC at Work, Leaders Guide.*
10. Kansas Mathematics Standards (CCSS). (2012). http://www.ksde.org/Default.aspx?tabid=4754
11. Killion, J. (2002). *Assessing impact: Evaluating staff development.* Oxford, OH: National Staff Development Council.
12. Learning Forward. (2011). *Standards for Professional Learning.*
13. Learn NC. http://www.learnnc.org/lp/editions/ccss2010-mathematics.
14. Learn Zillion. http://learnzillion.com/.
15. Mathematics Assessment Project. http://map.mathshell.org/materials/index.php.
16. McCallum, W., Daro, P., & Zimba, J. *Progressions Documents for the Common Core Math Standards.* Retrieved March 11, 2013, from *http://ime.math.arizona.edu/progressions/#about* .
17. Mid-continent Research for Education and Learning. (2000). *Principles in action: Stories of award-winning professional development* [video]. Aurora, CO: Author.
18. National Council of Teachers of Mathematics. (1991). *Professional Standards for Teaching Mathematics.*
19. National Council of Teachers of Mathematics. (2000). *Principles & Standards for School Mathematics.*
20. National Council of Teachers of Mathematics. (2006). *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics: A Quest for Coherence.*
21. National Council of Teachers of Mathematics. (2009) *Focus in High School Mathematics: Reasoning and Sense Making.*
22. National Council of Teachers of Mathematics. *Core Math Tools. http://www.nctm.org/resources/content.aspx?id=32702.*
23. National Council of Teachers of Mathematics. *Illuminations. http://illuminations.nctm.org/.*
24. National Mathematics Advisory Panel. (2008). *The Final Report of the National Mathematics Advisory Panel.* U.S. Department of Education.

25. National Staff Development Council. (2001). *Standards for staff development.* Oxford, OH: Author. Retrieved March 11, 2013, from http://www.nsdc.org/standards/index.cfm
26. Porter, A., Garet, M., Desimone, L., Yoon, K., & Birman, B. (2000). *Does professional development change teaching practice? Results from a three-year study.* Washington, DC: U.S. Department of Education. Retrieved March 11, 2013, from http://www.ed.gov/rschstat/eval/teaching/epdp/report.pdf
27. South Dakota Department of Education. *Curation Review Rubric for Mathematics Learning Resources.* Retrieved March 11, 2013, from http://www.livebinders.com/play/play?id=367846
28. Steiner, L. (2004). *Designing effective professional development experiences: What do we know?* Naperville, IL: Learning Point Associates.
29. Tools for the Common Core Standards. http://commoncoretools.me/.
30. Wisconsin Department of Public Instruction. (2010) *Characteristics of High-Quality and Evidence-Based Professional Learning*. Retrieved March 11, 2013 from, http://www.betterhighschools.org/MidwestSIG/documents/Rasmussen_Handout1.pdf
31. Yoon, K. S., Duncan T., Lee, S. W.-Y., Scarloss, B., & Shapley, K. L. (2007). *Reviewing the evidence on how teacher professional development affects student achievement*. (Issues & Answers Report, REL 2007–No. 033). Washington, DC: National Center for Education Evaluation and Regional Assistance, Regional Education Laboratory Southwest. Retrieved March 11, 2013, from http://ies.ed.gov/
32. Publishers Criteria: www.corestandards.org
33. Focus by Grade Level, Content Emphases by Jason Zimba: http://achievethecore.org/page/774/focus-by-grade-level
34. Georgie Frameworks: https://www.georgiastandards.org/Standards/Pages/BrowseStandards/MathStandards9-12.aspx
35. engageNY Modules: http://www.engageny.org/mathematicshttp://www.engageny.org/mathematics